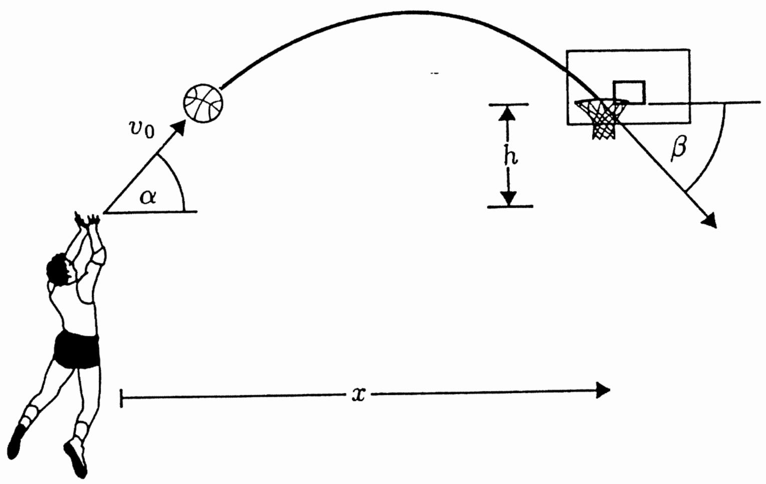
MATHEMATICS AND COMPUTERS IN SPORT



A Conference held at Bond University, Queensland, Australia 13th to 15th July, 1992

Sponsored jointly by

The Australian Mathematical Society

and

The Australian Sports Commission

Edited by Neville de Mestre, Associate Professor of Mathematics
School of Information and Computing Sciences, Bond University.

Vol 1

CONTENTS (Vol. 1)

PROGRAM

Monday 13 July

9.00am	John Croucher (Macquarie) - "Winning with science".
10.00 am	David Hoffman (ADFA) - "A team rating system".
10.40 am	*Morning Tea*
11.10 am	John Norman (Sheffield) - "Strategy in fell running : An analysis of the Bob Graham round".
11.50 am	Raymond Stefani (California State, Long Beach) - "Applying least squares to team sports and Olympic winning performances".
12.30 pm	*Lunch*
2.00 pm	Workshop on special problems
	Problem 1 : A timetable for entrants in equestrian events Problem 2 : Team ratings Problem 3 : The mathematics and physics of body surfing
3.30 pm	*Afternoon Tea*
4.00 pm	Workshop continued

Tuesday 14 July

9.00 am	Jon Patrick (Deakin) - "The marriage of mathematics and computer technologies for sport improvement".
10.00 am	Mark Johnston (Swinburne) - "An analysis of scoring policies in one day cricket".
10.40 am	*Morning Tea*
11.10 am	Stephen Clarke (Swinburne) - "Computer and human tipping of AFL football - a comparison of 1991 results".
11.50 am	Robert Neal (Queensland) - "A mathematical model of giant swings on the horizontal bar".
12.30 pm	*Lunch*
2.00 pm	Workshop continued
3.30 pm	*Afternoon Tea*
4.00 pm	Report on special problems

Wednesday 15 July

9.00 am	David Hoffman (ADFA) - "A taxonomy of sporting events applying operations research methodology".
10.00 am	Maurie Brearley (Clifton Springs) - "Hang gliders, wobbles and myths".
10.40 am	*Morning Tea*
11.10 am	Rod Weber (ADFA) - "The flight of a football"
11.40 am	Neville de Mestre (Bond) - "Mathematics applied to sport"
12.30 pm	*Close and lunch*

GENERAL

This is the first conference held in Australia on the relationship of mathematics and computers to sport. Its principal aim is to bring together researchers in these aspects of sport science, so that we will all be more aware of what research is being done, who is doing it, and where we can turn for an exchange of ideas or help on various topics.

There is no doubt that mathematics and computers are an integral part of our culture and our society. Mathematics has always been perceived by many as difficult, but necessary in scientific contexts. It has recently blossomed so that its applications are in many other disciplines besides science and engineering. Computers are a newcomer to the investigative and cultural scene. Originally closely linked with mathematics, there has been recently a tendency for the discipline of computer science to develop independently of mathematics. In some cases the pendulum has swung too far, and the benefits of using both mathematics and computing in models of some aspects of life have been completely overlooked. In the area of sports science I hope that the occurrence of this conference will help to reverse that trend, so that a useful balance is again established between the two disciplines.

I would like to thank the three invited speakers – David Hoffman (ADFA), Jon Patrick (Deakin) and John Croucher (Macquarie) for being willing to attend the conference and lead discussion. I would also like to thank all the participants for making the special effort to attend in these recession-hit days.

Finally I must place on paper my gratitude to my secretary, Lyn Hathaway, who has been an indispensable help in preparing this record of the proceedings and in helping me with the organisation.

Neville de Mestre
Conference Director
July 1992

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WINNING WITH SCIENCE

John S. Croucher[1]

Abstract

The use of scientific method in sport is not new. For many years scientists, statisticians, mathematicians and just plain sport enthusiasts have been collecting data with the hope that it will provide some magic formula to increase the chance of success of individuals or teams in sport.

The advent of computers has been of immeasurable value in this regard with modern machines now being able to process enormous amounts of information at the touch of a key. The question is just how useful all this research really is when translated into practical terms.

The number of sports using computerised statistics is seemingly endless and examples include all codes of football, hockey, basketball, water polo and baseball. Without such evidence players and coaches would have to rely on their own impressions and prejudices in developing their game plans.

It seems a fair comment that the scientific analysis of sports performance has not gained the acceptance it should have in Australia. This paper examines some of the reasons why and looks at the work done in several notable sports including tennis, netball and cricket. It also considers the question of just who really benefits from all this research.

1. Introduction

The use of scientific method in sports has existed in some form or another for over fifty years but it was not until the advent of computers in the

[1] Associate Professor of Statistics, Macquarie University N.S.W.

1960's that this field really began to blossom. It seemed for the first time that scientists with an interest in sport had an opportunity at last to collect mountains of data and 'run it through the computer' to hopefully come up with a winning strategy.

In the late 1970's and early 1980's the personal computer afforded even greater opportunity for the enthusiast who could now 'lug' their favourite machine to the sporting event itself and enter data as it happened. The subsequent advent of the laptop computer and lighter printers made this even easier and the sports scientist could now not only collect data as the event actually took place but analyse and print out the results as well.

There are many types of sports research, but a few that come to mind are:

1. The use of sophisticated statistical techniques to predict future world records.
2. Setting up a particular sport as a mathematical model.
3. Considering the effect of rule changes.
4. Analysing the data from a game that has already been played. The objective here is to identify the strengths and weaknesses of both yourself and your opponent.
5. Simulation of sporting events using probability distributions based on historical data.
6. Using statistics to explain unusual events such as Bob Beamon's extraordinary long jump at the 1968 Olympic Games in Mexico City.
7. Using statistics to detect remarkable and unexpected improvements in an athlete's performance with a view to determining if drug taking may be an issue.
8. Determination of whether world records have been affected by the recent introduction of drug testing.
9. The physical and psychological effects on athletes of performing in their sport.
10. Using statistics to consider the performance of referees and umpires, especially with regard to their supposed impartiality.

These are but some of the ways in which sport scientists have occupied themselves and this list is by no means exhaustive. Much of the work has indeed had practical application but there is also a great deal of investigation which, while 'interesting in itself' has proven of little or no real value to the competitor in actually playing the game.

The difficulty, however, may not be in the actual research. Many findings are indeed of great value but that in itself is often not enough. One of the major problems lies in 'selling' that idea to either a sporting body or coach and all over the world this does not seem always to be an easy matter.

Perhaps with some justification, people are naturally suspicious of things which they do not fully understand. Unfortunately, in sporting circles, these 'things' often include academics, statisticians, computer scientists, and, especially computers. (It's even worse when the researcher is some combination of these professions!)

It's one thing to have discovered some wonderful technique which you feel will revolutionise the way in which a particular sport is played - it's another to actually get the hierarchy in that sport to believe you. Another universal difficulty seems to be that academics themselves are very suspicious of research into sport. They feel that universities have no business wasting their time and resources with such trivia when there is much more important and serious (unspecified) work to be done. (The author hastens to add that he has personally been on the receiving end of such comments over the years.)

It is therefore with some trepidation that the academic embarks on a course of sports research. Research grants are not always readily forthcoming for such activities and even if the rewards sometimes seem to be few it seems to be only the truly dedicated sports fan who will throw caution to the wind and proceed regardless.

But what of the end result? Is it of any real benefit to obtain a theoretical result that could not possibly be of use to anybody playing the sport? For example, is it really important to learn that distribution of goals scored in soccer follows a negative binomial distribution or that the strategy that you have devised depends upon some statistical assumption which is probably not true in practice? Try telling that to the players and coaches and see how far you get!

But this doesn't mean that such research is not worthwhile - it's just that it runs the risk of languishing in some scientific journal without going much further. The prediction of future world records is another favourite. It's fascinating to look at all the techniques used to tell us what the world swimming, athletic and other records will be in fifty or even one hundred years time. It's a fairly safe thing to do - the odds are that the prediction will long be forgotten by the time the actual year rolls around anyway.

An offshoot of the aforementioned research is the issue of whether there is any current world record which may never be broken. For twenty three years many sport scientists were declaring that Bob Beamon's long jump was the one record that would never be broken. Ironically, it was broken by two different men within 30 minutes of each other at the same athletics meeting. Interestingly, people are no longer saying that this new record is unbreakable - perhaps it just isn't the statistical anomaly of its predecessor.

So where do we go from here? The sports scientist clearly not only has to be good at his research but in convincing people that it is all worthwhile. They must be able to point at previous work and be able to say that the results gained enabled players to actually benefit from the result. In particular, they might be able to refer to leading professionals who use such results.

An example of this was highlighted to the author on a recent trip to the U.K. where a salesman had (through much research) invented a device which would allegedly help anybody playing a particular sport to improve dramatically. On paper the idea looked good although the device itself was not cheap. The problem was that the salesman could not name **any** well known player of the sport who actually used the device and that in itself placed doubt in any prospective purchaser's mind.

How then does one get a good idea off the ground? The author has found (from hard experience) that it is rarely of much use sitting in your office coming up with allegedly wonderful ideas as to how a certain sport could either be improved or played better. If one is really serious about having an idea adopted then it is highly recommended that the relevant sporting body be put onside right from the outset. Speak to the relevant state directors of coaching, team coaches, players (at all levels) and managers. Talk about your proposed ideas and ask them for an opinion or any suggestions or improvements that they might make.

In other words, make them part of the research. You'll soon get the message if you are likely to encounter resistance to the whole project at the end of the day. The more involved they become as the research proceeds the less likely it is that they will be able to pour cold water on it at the end. Their involvement doesn't necessarily mean acceptance of your findings but it usually guarantees at least a fair hearing!

There have been a number of instances over the years of where data recording of certain sports has actually led to changes in how the game is played. The following sections discusses some of the successes but sadly, for every one of these there is surely many ideas which have never been, and probably never will be, adopted.

2. Tennis

The sport of tennis has long been a favourite with researchers because of the very nature of the discrete events which are part of its make-up. Typical research has included the consequence of rule changes (Croucher [2]), modelling the scoring system (Miles [8], Schutz[10]), serving strategies (George [6]), an analysis of the tie-breaker (Croucher[3], Pollard[9]) the energetics of singles tennis (Elliott[5]), the flight of a tennis ball (Jones [7]), conditional probabilities of winning a game (Croucher[4]) and a complete mathematical analysis (Carter[1])

It is hard to quantify the impact such papers have had on the game - the rules haven't changed in recent years and most probably no professional tennis player could quote any of these works. But having said that, there is one piece of ingenuity which does seem to have had an effect on at least some of the top players.

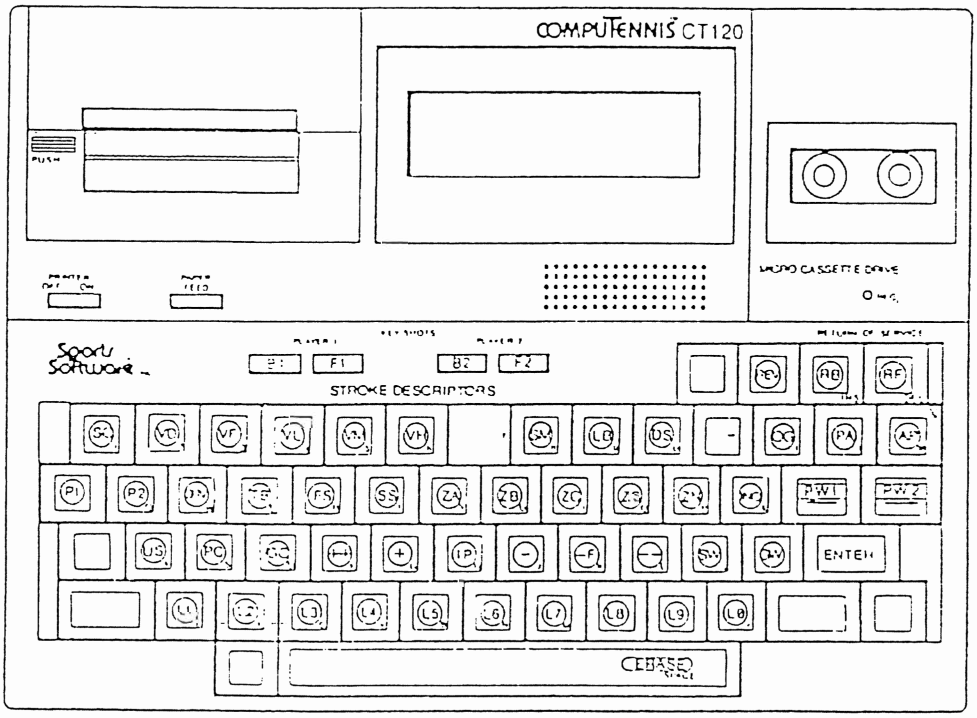
It all began late in 1982 when a South African amateur tennis player named Bill Jacobsen began charting his son's tennis matches in the U.S.A. He initially recorded the information with pen and paper but found that process quite slow and cumbersome. Jacobsen actually designed a small 2 kg. computer (about the size of an A4 paper) and which was charged by rechargeable batteries and had an inbuilt printer. (See Figure 1.) He found in this way he could record up to ten times the data than previously.

The technique, which became known as CompuTennis and was well on its way to widespread acceptance when the Stanford tennis team used it to produce their match reports and in June 1984 the service was given a contract to chart matches for all U.S. national junior teams.

The data is fed into the computer by one of three ways:

1. At courtside whilst play is in progress
2. From a video tape of the match
3. Hand charting using CompuTennis hand charts

Figure 1. The CompuTennis computer



Each key on the keyboard is programmed for a particular function which are divided into five categories:

1. The shot type
2. The stroke description
3. The direction of the shot
4. The result of the shot
5. The location of each player at the end of each point.

Not all strokes are included - only those that influence how a point is won or lost. Output is given in three different ways:

1. Printed using the computer's micro printer
2. Around 40 different statistics for each player can be quickly displayed on the screen at any time
3. Full reports may be obtained using an external printer.

The whole idea behind CompuTennis is that it readily identifies the strengths and weaknesses of players. This includes not only their individual strokes but combinations of strokes. A typical match report is shown in Table 1 for a match played between two Australian ladies Potter and Suttie.

Among some of the more interesting of the earlier findings of CompuTennis are:

1. Jimmy Conners easily had the best return of serve of any professional. His percentage of points won was consistently 10% to 20% higher than the others.

2. Winners among juniors are usually defensive players who make few mistakes. However, by age 18, it is the more aggressive players who rise to the top.

3. Among the professionals the superior players are usually the ones with the more reliable backhands. The better male players have significantly fewer backhand errors.

4. In women's tennis the ability to serve strongly is critical. The victors serve 25% more winners and aces than the players they beat. In men's tennis the difference is only 10%.

5. Apparently Chris Evert Lloyd, for example, despite her reluctance to come to the net, won 15% more points when she did so. As a result she changed her game by coming in more often.

The concept has apparently caught on in a big way all over the world and the CompuTennis people claim to have many of the top professionals using the system and to have charted many thousands of matches. The author is aware of at least 1 researcher in Koln, Germany, who is working on the development of a similar system for doubles.

Table 1. An example of CompuTennis output

C O U R T   M A T C H   S T A T I S T I C S

MATCH DATE: 04/12/86 LOCATION: WAGGA
FOTTER BEAT SUTTIE SCORE: 6-3 6-3
T/BS:

COMPLETE MATCH		FOTTER AUSSIE		SUTTIE AUSSIE	
1. SERVICE					
1st SERVES IN PLAY	(%)	36	(76.6%)	33	(51.6%
1st SERVE POINTS	WON:LOST (%)	27:9	(75.0%)	21:12	(63.6%
DEUCE COURT POINTS	WON:LOST (%)	16:5	(76.2%)	9:8	(52.9%
AD COURT POINTS	WON:LOST (%)	11:4	(73.3%)	12:4	(75.0%
2nd SERVE POINTS	WON:LOST (%)	6:5	(54.5%)	11:20	(35.5%
DEUCE COURT POINTS	WON:LOST (%)	3:2	(60.0%)	3:13	(18.8%
AD COURT POINTS	WON:LOST (%)	3:3	(50.0%)	8:7	(53.3%
ACES AND SERVICE WINNERS		5		0	
DOUBLE FAULTS		1		4	
SERVICE WIN:ERROR RATIO		5:1	(5.00)	0:4	(0.00
2. RETURN OF SERVES					
1st SERVE RETURN POINTS	WON:LOST (%)	12:21	(36.4%)	9:27	(25.0%
2nd SERVE RETURN POINTS	WON:LOST (%)	20:11	(64.5%)	5:6	(45.5%
FOREHAND RETURNS IN PLAY:ERRORS	(%)	31:5	(86.1%)	7:9	(43.8%
BACKHAND RETURNS IN PLAY:ERRORS	(%)	21:3	(87.5%)	20:5	(80.0%
FORCING RETURNS	(%)	13	(21.7%)	3	(7.3%
RETURN WIN:ERROR RATIO		1:8	(0.12)	1:14	(0.07
3. GROUND STROKE – KEY SHOTS					
FOREHAND WIN:ERROR RATIO		11:16	(0.68)	6:16	(0.37
FOREHAND UNFORCED ERRORS		8		4	
BACKHAND WIN:ERROR RATIO		4:8	(0.50)	2:10	(0.20
BACKHAND UNFORCED ERRORS		5		4	
ALL GRND STROKES WIN:ERROR RATIO		15:25	(0.60)	8:26	(0.30
BACKCOURT POINTS	WON:LOST (%)	40:34	(54.1%)	30:39	(43.5%
4. NET PLAY – KEY SHOTS					
POINTS ADVANCING TO NET	NUMBER (%)	3	(2.7%)	8	(7.2%
NET POINTS	WON:LOST (%)	1:2	(33.3%)	6:2	(75.0%
FOREHAND VOLLEY WIN:ERROR RATIO		0:1	(0.00)	1:0	(----
BACKHAND VOLLEY WIN:ERROR RATIO		0:0	(0.00)	0:0	(0.00
VOLLEY UNFORCED ERRORS		1		0	
ALL NET PLAY WIN:ERROR RATIO		0:1	(0.00)	2:0	(----
5. SUMMARY					
OVERALL WIN:ERROR RATIO		21:35	(0.60)	11:44	(0.25
UNFORCED ERRORS	NUMBER (%)	18	(16.2%)	15	(13.5%
PTS WON W/ FORCING SHOTS	NUMBER (%)	50	(45.0%)	28	(25.2%
TOTAL POINTS	WON:LOST	65:46		46:65	

Are there any down sides to such a system? A number of coaches have expressed grave reservations about the heavy reliance on statistics. Some feel that statisticians analyse data too critically while others contend that there is a danger that the negative aspects of a performance will be the ones highlighted. This can result in players becoming tense before a major event on the grounds that 'Big Brother' will be watching them.

In many instances, on an individual basis, a sports psychologist can assess whether these types of systems are doing more harm than good as the emotional state of a player is also very important.

2. Netball

In 1987, with the assistance of a grant from the N.S.W. Department of Sport and Recreation, a system for recording and analysing information from netball games was developed by the author. The game of netball is easily the most participated sport by females in Australia with over 800,000 registered players.

The concept was named NETSTAT and consists of several phases:

1. A coding sheet for match information.
2. A recording sheet for match data.
3. Entering of match data into a computer program.
4. Obtaining match reports.

The development of the system took about eighteen months and much of the time was spent talking to coaches at all levels and designing and redesigning the recording sheet. Opinions about what aspects of the game to include were sought from officials and players until finally there was a system which seemed to incorporate most of their wishes.

NETSTAT was trialled on numerous games at state levels in N.S.W. and then on a number of international matches including Australia v Trinidad. It was also used by the Australian team at the 1990 Commonwealth Games in New Zealand where Australia won the gold medal. The current N.S.W. Director of netball coaching regularly uses NETSTAT for her own team in the Australian netball Super League.

But the battle is not yet completely won - the idea of a sophisticated computerised system is still very radical to many of those involved in the sport and the resistance to change is quite apparent. Nevertheless, the benefits of NETSTAT, a world first in its sport, have already been felt and the author believes that it will eventually win the day!

Because of the amount of information required and the speed of the game, two people are usually used for recording match data if doing so from a live game.

The purpose of NETSTAT was to provide a scientific analysis of the game of netball. The details provided by this system will enable coaches to have an efficient and comprehensive analysis which would include the following benefits :

1. The records of individual players or even the entire history of a netball competition can be recalled at any time.
2. Reasons for success and failure can be easily obtained from past performances.
3. Strengths and weaknesses of individual players will become readily apparent.
4. Methods for successful performances can be developed.
5. Decisions can be made objectively from factual information.

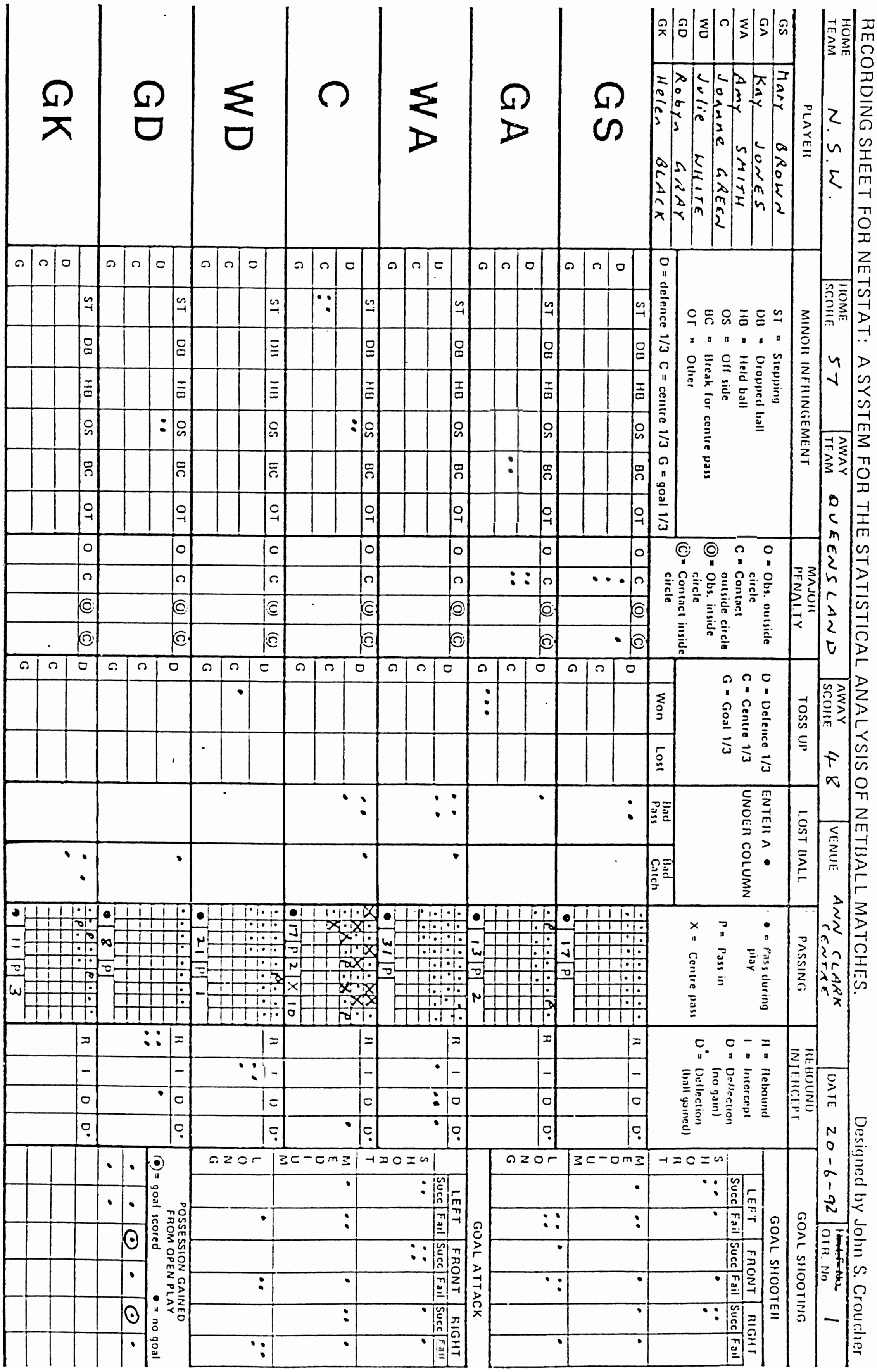
In order to provide an extremely detailed look at all aspects of play in a game, relevant pieces of data are entered on a recording sheet (as shown in Table 2). A separate recording sheet is required for each half (or quarter) in a match.

Since netball is a fast moving game it is generally not possible for one person to record the data alone while a match is in progress. Experience has shown that a good arrangement is to have two people involved - one of these calls out what is happening while the other does the recording. In fact, the person recording the data does not even have to look at the play !

The person who calls out the action should, of course, be thoroughly familiar with the rules of netball. In particular, they should be able to interpret any decision that an umpire may make and be able to identify the position of any player involved.

If match is recorded on video then it should be possible for one person to fill in the recording sheets from a video replay. However, a lot will depend on the position of the camera in determining, for example, which player may have incurred a penalty or thrown a bad pass.

Table 2. The NETSTAT Recording Sheet



One advantage in recording a match live is that a coach is then able to use the information already gathered as the match progresses. The data may lead to positional changes, replacement of players or an alteration of tactics.

The decision as to which statistics would be of most benefit to a coach is always going to be debatable but those provided by NETSTAT should satisfy most. Briefly, the information recorded includes :

1. Match information (such as team names, scores, venue, date)
2. Player names
3. Minor infringements
4. Major penalties
5. Results of toss ups
6. Lost balls (via a bad pass or bad catch)
7. Passing (how many times each player has passed the ball)
8. The number of rebounds, deflections and interceptions made
9. Full goal shooting details (including the position and length of the shot, whether it was successful and whether it was the result of a penalty)
10. Possession gained from open play (whether a team was able to convert an unexpected gain of the ball into a goal)
11. Any special comments

Once all of the information on a match has been recorded the next step is to enter the data into a specially designed computer program. Written in the turbo Pascal language, this is a very sophisticated program which can analyse the data and provide those details which are invaluable to coaches.

The program has been written so that the data from the recording sheets can be easily entered. In fact, the recording sheets and the computer program were designed simultaneously in order to compliment each other and to have a similar appearance.

When using the program there are a number of types of reports that can be specified. For example, only a brief summary of the entire match statistics might be required. On the other hand, a full detailed half-by-half (or quarter-by-quarter) analysis can be obtained for each individual player. It is easy for the user to specify what option is required and whether the information is to appear only on the terminal screen or is to be sent to a printer.

Each piece of information is entered onto the recording sheet. Because of limited space on the sheet some explanations of the codes have been abbreviated. Since netball is a fast-moving game, any recording of data must be done as quickly as possible. For this reason, the majority of information recorded is done by simply marking a 'dot' in the appropriate position on the sheet. The only exceptions are for certain types of passes and for comments.

The **match information sheet** is used for identification of the match. This data includes the home team, home score, away team, away score, venue, whether the match is played in halves or quarters and the date. While it does not matter which team is nominated as the 'home' team for recording purposes, note that the computer program for NETSTAT expects that the details on the sheet will be those for the **home** team.

There is also space for the players' names to be entered, preferably with a given name (or initial) followed by surname. These should be written next to their corresponding position for that half (or quarter). The full position titles are:

GS = goal shooter
GA = goal attack
WA = wing attack
C = centre
WD = wing defence
GD = goal defence
GK = goal keeper

There are a number of minor infringements that can occur, but many of these happen so rarely that it is not necessary to list them all for this purpose. Five of the most important ones have been identified and all others have been simply included under the heading 'other'.

The six types considered and their codes are

ST = stepping
DB = dropped ball
HB = held ball
OS = off side
BC = break for centre pass

OT = other technical infringement

Any such infringement is recorded by code according to the player involved and the position on the court where it occurred. It is considered important by many coaches to be able to identify just where these problems are occurring. To this end, the court is divided into three equal areas identified as :

D = defensive 1/3 of court
C = centre 1/3 of court
G = goal 1/3 of court

For example, if the wing defence infringed by 'stepping' in the centre 1/3 section of the court, then a dot would be recorded in the box next to WD in the row labelled C and the column headed ST. Also, if the goal attack 'held the ball' in the goal 1/3 of the court then a dot would be recorded in the box next to GA in the row labelled G and the column headed HB.

There is no limit on the number of dots that may be recorded in any box.

An example of the sort of thing that the caller of the play might say (in order) to the recorder in these cases is :

'Minor infringement, wing defence, centre 1/3, stepping', or
'Minor infringement, goal attack, goal 1/3, held ball'

The **major** penalties include contact and obstruction and whether they occurred inside or outside the circle. The specific codes are:

O = obstruction outside the circle
C = contact outside the circle
Ⓞ = obstruction inside the circle
Ⓒ = contact inside the circle

The symbols used in these codes are designed to make them easier to remember. In this case dots are entered under the appropriate column in the same row as the player who incurred the penalty.

For example, if the goal keeper 'contacted outside the circle' then a dot would be entered in the column headed C and in the row corresponding to GK.

In this case the caller might say :

'Goal keeper, contact, outside circle'

There is no limit on the number of dots that may be recorded in any box.

For the **toss ups** the things of interest are which player was involved, which 1/3 of the court it took place and whether it was won or lost. The codes are:

D = toss up took place in defensive 1/3
C = toss up took place in centre 1/3
G = toss up took place in goal 1/3

The appropriate code is entered in the row corresponding to the player and the appropriate part of the court (D, C or G) and under the Won or Lost column as the case may be. For example, suppose that the goal attack lost a toss up in the goal 1/3. Then a dot would be entered in the Lost column in the row labelled G corresponding to GA.

In this case the caller might say

'Toss up, goal 1/3, goal attack, lost'

There is no limit on the number of dots that may be recorded in any box.

These statistics are recorded whenever a team loses possession either by a bad pass being thrown or a bad attempt at a catch being made. It is the one statistic in which the recorders have to make a subjective judgement - that is, whether it the fault of the passer or the catcher. In some cases it may even be both !

In this case no codes are required. All that has to be done is to enter a dot under the appropriate column for the player concerned.

For example, if the centre threw a 'bad pass' then a dot would be recorded under the Bad Pass column next to the row corresponding to C.

In this case the caller might say:

'lost ball, bad pass, centre'

There is no limit on the number of dots that may be recorded in any box.

The **passing statistic** is the one which is perhaps the most difficult to record and which requires a degree of concentration. It essentially involves the recording of **every** pass thrown by the home team according to who threw the pass. Passes are classified into three different types by the following codes:

• = pass made during the course of ordinary play

X = centre pass (made by the centre to resume play)

P = pass thrown in from the sideline

These codes may be entered in any order within the appropriate squares but it is recommended that you begin in the top left hand corner and work **across** the rows.

For example, if the wing attack threw a pass during play then a • is entered in one of the small squares in the PASSING column in the row corresponding to WA.

For a typical sequence of play, the caller of the play may say:

'centre pass to wing attack to centre to wing defence to centre to goal attack to goal shooter who shoots for goal'

Note that there is space on the sheet to record up to 54 passes per player per half (or quarter). In the rare case that this number is exceeded for any player more than dot may be entered in the little squares. Normally, however, only one dot should be recorded per little square since this makes addition of the dots much easier.

The bottom row for each player provides a space for totalling the various types of passes made. For all players except the centre there is space for totalling the dots and the P's. For the centre there is also space for totalling the X's. These totals should be entered after the period is completed.

The **rebound/intercept** statistics record whether a player has been able to take a rebound or to intercept a pass thrown between two players in the opposing team. Also recorded is whether a player has been able to touch (or deflect) a pass between opposition team members without gaining possession or whether they have been able to deflect a pass from the opposition so that possession is gained by another member of their team.

The four codes used are :

R = rebound taken

opposition

I = an intercept is made of a pass between two team members

D = ball is deflected (or touched) when the opposition is passing but possession was not gained

D* = ball is deflected (or touched) when the opposition is passing to the extent that another of the home team was able to gain possession.

For example, if the wing defence was able to intercept a pass then a dot would be recorded in the column under I in the row corresponding to WD.

In this case the caller might say :

'intercept, wing defence' and then proceed to call the next sequence of passes to be entered in the PASSING column.

There is no limit on the number of dots that may be recorded in any box.

Comprehensive **goal shooting** statistics are recorded including which player took the shot, the length and position of the shot and whether or not the attempt was successful.

The caller of the play will have to decide on the length of a shot. A rule of thumb would be that a shot taken adjacent to the post would be 'short' and one from the edge or near edge of the circle would be 'long'. Most other attempts would be 'medium'.

It is also recorded from which **side** of the post the attempt was made. That is, if it was made from the left hand side of the post, in front of the post or from the right hand side of the post. For each shot attempted a dot is recorded in the appropriate box. (It may be informative to enter a P instead of a dot if the attempt was as a result of a penalty. However, the computer program will just interpret a P as a dot.)

Suppose during play that the goal shooter took a long range shot (from the edge of the circle) in front of the post and missed. Then, in the GOAL SHOOTER section, a dot would be entered under the Fail column of FRONT in the row labelled LONG.

In this case the caller might say :

'goal attempt by goal shooter, long, front, fail'

There is no limit on the number of dots that may be recorded in any box.

One of the concerns of many coaches is the number of times that their team is able to convert 'unexpected possession' during play into a goal. This possession typically comes from a mistake by the opposition such as their throwing a bad pass or making a bad catch.

In the bottom right hand corner of the scoring sheet there are boxes into which may be entered the appropriate code. The individual players involved are not recorded. The codes are :

⊙ = unexpected possession converted into a goal
• = unexpected possession not converted into a goal

For example, suppose that the wing attack intercepted an opposition pass. A code of I would be recorded in the REBOUND/INTERCEPT column in the WA column and at the same time a would be noted in the POSSESSION GAINED FROM OPEN PLAY section. If at the end of the passing sequence a goal was scored, then a circle would be drawn around the • making it look like ⊙. If a goal was **not** subsequently scored then the • would be left as it is.

On the back of each scoring sheet is a space for **comments**. After each half (or quarter) is played the recorder is able to make special notes of anything unusual that occurred during that period. Such comments could include weather conditions, positional changes, replacement of players (from either team,) any injuries sustained or notes on tactics. The computer program is able to incorporate any comments that are recorded.

The most demanding task in entering the information onto the recording sheet is undoubtedly the PASSING column. This is due to the speed at which the passes are thrown and is the main reason for two recorders being required.

If, however, it is desired that the passes **not** be recorded, then it should be quite possible for **one** person to be able to record the rest of the information on the sheet.

Table 3 shows a brief team summary of the match in Table 2. The match is a hypothetical one between N.S.W. and Queensland played at the

Ann Clark Centre in Sydney on 20 June 1992. N.S.W. won the game by 57-48 and the scoring sheet is for the first of four quarters.

Table 3. Brief team summary give by NETSTAT

***** NETSTAT *****

***** BRIEF TEAM SUMMARY *****

Home team : N.S.W.
Away team : Queensland
Home score : 57
Away score : 48
Venue : Ann Clark Centre
Date : 20/06/92

	GS	GA	WA	C	WD	GD	GK	TOTAL
Minor infringements	1	3		7	1	3		15
Contact in circle	2	1				1	3	7
Contact outside circle	5	6						11
Obstruction out circle				1			1	2
Obstruction in circle								
Toss up won		4			2		2	8
Toss up lost		1			1		2	4
Bad pass	3	2	5	4			1	15
Bad catch			2	1		4		7
Passes during play	62	46	97	85	60	28	19	397
Centre pass								37
Pass in	4	2	1	2	2	2	8	21
Rebound	2	3				6	2	13
Interception	1	1				4		6
Deflection (gain)	2			3			1	6
Deflection (no gain)	1	2		2	1	4		10

It is up to the individual player or coach to interpret the output, but some results seem self-evident even after the first quarter.

1. The goal attack has twice broken for the centre pass.

2. The centre has stepped three times in centre court.

3. The centre and goal defence have each twice been off side in the defensive court.
4. The goal shooter has contacted three times and the goal attack four times outside the circle.
5. The goal attack has won all three toss-ups in the goal court.
6. The wing attack has thrown four bad passes and the centre has thrown three. Each time possession has been lost.
7. The goal keeper has lost possession with three bad catches.
8. The goal defence has made four rebounds.
9. The wing defence has made three intercepts.
10. The goal shooter has had 13 successes and 11 fails. She has shot poorly to the left of the post (4/11) but well from the right (8/10). She has made 6/8 short shots but only 2/7 long shots.
11. The goal attack has had 16 successes and 6 fails. In particular, she has not only made 7/8 short shots but 6/7 long shots.
12. The team was only able to convert two out of eight possessions from open play into goals.

COMMENTS

The match commenced at 1.00 pm. on Saturday afternoon. The weather was overcast and light drizzle fell at the start of play and sometimes throughout. About 150 spectators were present.

There are of course other observations that could be made but these give some idea of how a coach or player might devise tactics based on the data as it is gathered.

Match reports on individual players are also an option provided by NETSTAT as well as a full breakdown of goalshooting statistics.

4. Cricket

In a similar vein the author has also developed a computer system called CRICKET-STAT for analysing the game of cricket. It is not unlike NETSTAT in concept but on a much larger scale since cricket matches can sometimes go for five days or 30 hours.

CRICKET-STAT takes an extremely detailed look at every ball bowled. In particular, up to eleven pieces of information are recorded for each ball and hence in a typical day's play there would be 90 x 6 = 5940 pieces of data to be analysed.

This system is also quite unique and has been used in international matches as well as by the N.S.W. Cricket Association in Sheffield Shield games. The aim of CRICKET-STAT is to provide players and coaches with information to identify the strengths and weaknesses of both their own efforts and that of the opposition.

5. Other Sports

The list of other sports which could benefit from the type of foregoing analysis is endless. There is no doubt that the age of proper scientific analysis is well and truly with us and vast amounts of money will be spent by teams trying to gain that winning edge.

One of the more recent techniques involves the link up of video recorders and computers to create a data base directly from a videotape of the game. To date this has been quite successful in soccer and current work on this aspect is being undertaken, for example, at the Deutsche Sporthochschule in Köln, Germany.

There is, of course, a proper place for both the theoretician and practitioner in all of this research. But the public relations task of selling the results to an often sceptical audience cannot be taken lightly.

If the aim is to develop a system which will truly be of real benefit to a particular sport then there is much more work to be done than just simply obtaining the result itself.

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A TEAM RATING SYSTEM

DAVID HOFFMAN[1]

Abstract

In a period of increasing involvement in sport, many systems have been developed for rating individual performances. However few have addressed the problem of assessing individuals within a team performance. This paper describes a system that has been used successfully for a number of years to assess individuals' performances in duplicate bridge events. However the system is easily converted to other sports with similar competitive structures.

Introduction

From the earliest of times people have attempted to rate individuals and teams in a wide range of activities - from sporting endeavour of individuals to world domination by nations. In previous generations this often only involved determining who was the best in a particular field. However this century there has been a rapid increase in the desire of organisations to rank all those entities participating, due in part to the increased exposure via the news media, and also to the increasing ease with which statistical data can be gathered and analysed.

This is particularly true in the areas of sporting and recreational activities, since these activities tend to be appreciated by almost everybody, and often represent the focus of entertainment and recreational discussion by large sectors of the community.

One of the first sports to attempt to rank individuals was cricket, with detailed statistics being kept on all first class games in England from the middle of the last century. It is a commentary on the game that over a hundred years later, the measures used to describe performances of individual players, such as batting and bowling averages, total runs scored, and wickets captured, have not changed. In comparing individuals these measures are often quoted, but rarely used to resolve arguments about who was or is the best. This is because individual performances are affected by the caliber of both the opposition and teammates, as well as the characteristics of the playing fields on which the results were obtained. Even more notable is that in attempting to determine which team was best, very little agreement exists on how to measure this characteristic. Some attempts have been made to quantify the approach to ranking teams using techniques such as the analytic hierarchy approach [Sinuany-Stern]. However these often require subjective input that leaves any resultant ranking open to criticism.

It is still very much the norm that for activities involving competitions between teams any measures produced either consist of comparing teams which are in direct competition or involve individual performances disregarding the capabilities of others competing in the same competition. However by the very nature of team events, even within a single competition, the measure of a teams performance is tied to the individuals forming the team. Thus while a team may be regarded as the best in a competition, at any stage in the competition it may not be the best due to the combination of individuals chosen to represent the team.

While people like to compare the ability of teams, it is generally of secondary importance to the desire to compare the ability of the individuals that comprise these teams. As has been mentioned above, there are often statistics collected that can help in this procedure. However these statistics can only be used as a guide rather than a definitive statement, and then only about the particular characteristic being measured.

The work reported in this paper aims to develop a methodology that allows comparison between teams, whether they be in direct competition, or in different competitions, provided there is a link between the two competitions in the form of individuals participating in both competitions. In particular it allows the performance of the team to be reflected in the performance of each member of the team, taking into account the relative ability of the

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opposing team(s), and if necessary other factors, provided they can be rated in a meaningful fashion.

Ratings

In attempting to rank players a distinction must be drawn between a player's ability and their performance.

The ability of a person can be defined as their capability, compared to others competing in the same area. It represents a long term measure which should change slowly over time, based only on performances which are significantly different from what is expected from the person. As an example of this international chess players have ELO ratings assigned. These ratings are accepted as being consistent over time, which allows comparison of players from different generations, who may never have met over a chess board.

The performance of a person is a measure of their results, generally over a short period of time, but may also be applied over the career of the person. Since performance is determined from results against others, either in direct competition, or over a similar time span, it is useful in determining who is currently performing best but is inappropriate in allowing comparisons over different time intervals. Performance indicators generally can change rapidly based on the latest events conducted. In sports such as gymnastics, swimming and boxing, 'World Champions' are determined on the basis of a single event. Other sports such as golf and tennis produce monthly or quarterly ratings. While these are based on performance over the preceding year, consecutive lists exhibit significant changes for the majority of players, depending on which new events have been included and which old events have been discarded.

While the ability of a person is a more reliable measure, there are a number of reasons why many major organisations use a performance measure:

- The measure is used to offer places in forthcoming events. Organisations generally wish to include those players who are currently dominating their sport, to attract both spectators and sponsors.

- It is significantly easier to construct a system that rewards results, rather than considers a player's result compared to a player's expected result (ability). This is especially important in sports requiring large media exposure to maintain the level of sponsorship required for future survival.

While accepting the need for organizations to determine performance, it seems more appropriate that any scheme to rate individuals in team competitions or in most individual competitions should implement an ability driven system. However in attempting to implement any scheme it is important to attempt to satisfy the following:

- ease of use

- acceptability to those utilising the scheme.

Evolution

The rating scheme developed was an outgrowth of a collaborative grant awarded in 1980 between the Canberra College of Advance Education and the University of New South Wales, to develop the *"AUSTPORT" Rating Points: Feasibility and Pilot Study*. The initial activity to which the study was addressed was yachting and, by the end of 1981, the *AUSAIL Yacht Race Recording System* was developed. The system was successfully demonstrated operationally at the ACT Tasar Trials, the World Tasar Trials and the National 505 Trials during 1982.

Concurrent with this development a rating system was developed to be implemented for duplicate bridge. The system was prototyped during 1983 and introduced in the ACT at the commencement of 1984. During the first two years operations, minor modifications were made to the system on the recommendation of those involved with the system. Since its inception it has run concurrently with a previously implemented national scheme, the Masterpoint scheme.

The Masterpoint scheme, while satisfying some of the requirements to rate players, has a number of obvious weaknesses which has invalidated it as a reliable indicator of a player's strength and ability. It was to overcome these deficiencies, while using an existing scheme as a benchmark, that duplicate bridge was chosen as the recreation for which the the rating scheme was implemented.

Model

Let there be a database of players $X = \{x_i : i = 1, \ldots, n\}$ where n may be variable over time as new players are added to the system, and existing players depart the system.

For each player there is associated two values :

- a rating, r_i
- the number of events rated in the current period, e_i, where the period may be chosen to satisfy the nature of the environment (the current year for duplicate bridge).

For the set X it is assumed that the ratings are normally distributed with a mean of 1000, and a standard deviation of 300, implying that the range of ratings will fall between 0 and 2000.

For every event there will be a set of teams, k. Each team will consist of a number of players, m_j [$j = 1, \ldots, k$], with each team in the event having a result s_j.

Each team's rating, tj, is determined as the average of each member of the team. Hence

$$t_j = \sum_{i=1}^{m_j} r_i / m_j \qquad j = 1, \ldots, k$$

The basic assumption underlying the method is that the distribution of team ratings and team results satisfies the joint normal distribution, giving

$$f(S,T) \sim N(\mu_s, \sigma_s, \mu_t, \sigma_t, \rho)$$

where μ_s = the average of results s_j
σ_s = the standard deviation of results s_j
μ_t = the average of team ratings t_j
σ_t = the standard deviation of team ratings t_j
ρ = the correlation coefficient

From this distribution, the conditional distribution of results is

$$g(S \mid T) \sim N(\mu_j, \sigma_j \mid t_j)$$

where μ_j = the expected result for team j
σ_j = the expected standard deviation for tean j

Adjustments in players' ratings are then made based on the difference between the expected result and the actual result. To avoid unnecessary changes associated with natural fluctuations of actual results, adjustment in rating only occurs when the difference is above a reasonable threshold.

Implementation

As previously mentioned, the system has been implemented in the major duplicate bridge club in the ACT since 1983. The database consists of about 500 members, with over 50 events conducted each year which are suitable for rating purposes. These events can be classified into two groups:

- pairs events where each unit consists of two individuals
- teams events where each unit consists of from four to six individuals.

As well the sponsoring organization classifies each event, based on it's importance, assigning a weighting between 1 and 5. This weighting is included when adjusting ratings, as follows, where the threshold is set at 0.9:

Normalized result	Adjustment
1.9 to ∞	10 * W * [1 + (O - 1.9)/4]
0.9 to 1.9	10 * W * [O - 0.9]
-0.9 to 0.9	0
-1.9 to -0.9	10 * W * [O + 0.9]
-∞ to -1.9	10 * W * [1 + (O + 1.9)/4]

where O = normalized conditional result
W = event weighting

The means and standard deviations used for each event are determined from the event sample, while the correlation coefficient is set to 0.6. Results over the eight years are displayed in Table 1

Year	Pairs		Teams	
	Number	Correlation	Number	Correlation
1983	16	.557	7	.739
1984	31	.559	17	.632
1985	36	.481	16	.568
1986	33	.539	16	.584
1987	36	.519	18	.674
1988	48	.538	14	.696
1989	35	.549	18	.757
1990	46	.602	17	.771

Table 1

A number of special characteristics are included to satisfy the duplicate bridge implementation:

i) A bonus is given to each individual in a unit which achieves the aim of the event being conducted. Generally this is for winning, but is also given for units qualifying when that is the aim of the event.

ii) The number of times that an individual earns an increase in their rating in a given year is also accumulated in the player database. When a player achieves three increases, all subsequent increases incur a additional bonus. This modification to the system was incorporated in 1985 to allow new players, who were added to the database with an initial rating that was generally significantly lower than their actual ability suggested based on results achieved, to relatively quickly establish ratings consistent with their ability.

The effect of this last factor can be seen by considering the movement in average correlation over a year achieved between ratings and results, as shown in Figure 1. In the first two years operations, many active players were added to the database with starting ratings significantly below their ability, with the result that correlation between ratings and results became less reliable. However as the set of ratings shifted over time to better reflect the relative ability of players, there occurred a consequential improvement in the correlations achieved.

At the end of each year a shift in the set of ratings is performed to reinitialize the set to have a mean of 1000 with a standard deviation of 300. During this process, players who have not competed in at least eight percent of the available events have their ratings modified down. This ensures that inactivity cannot be used as a method of maintaining a players position in the ratings.

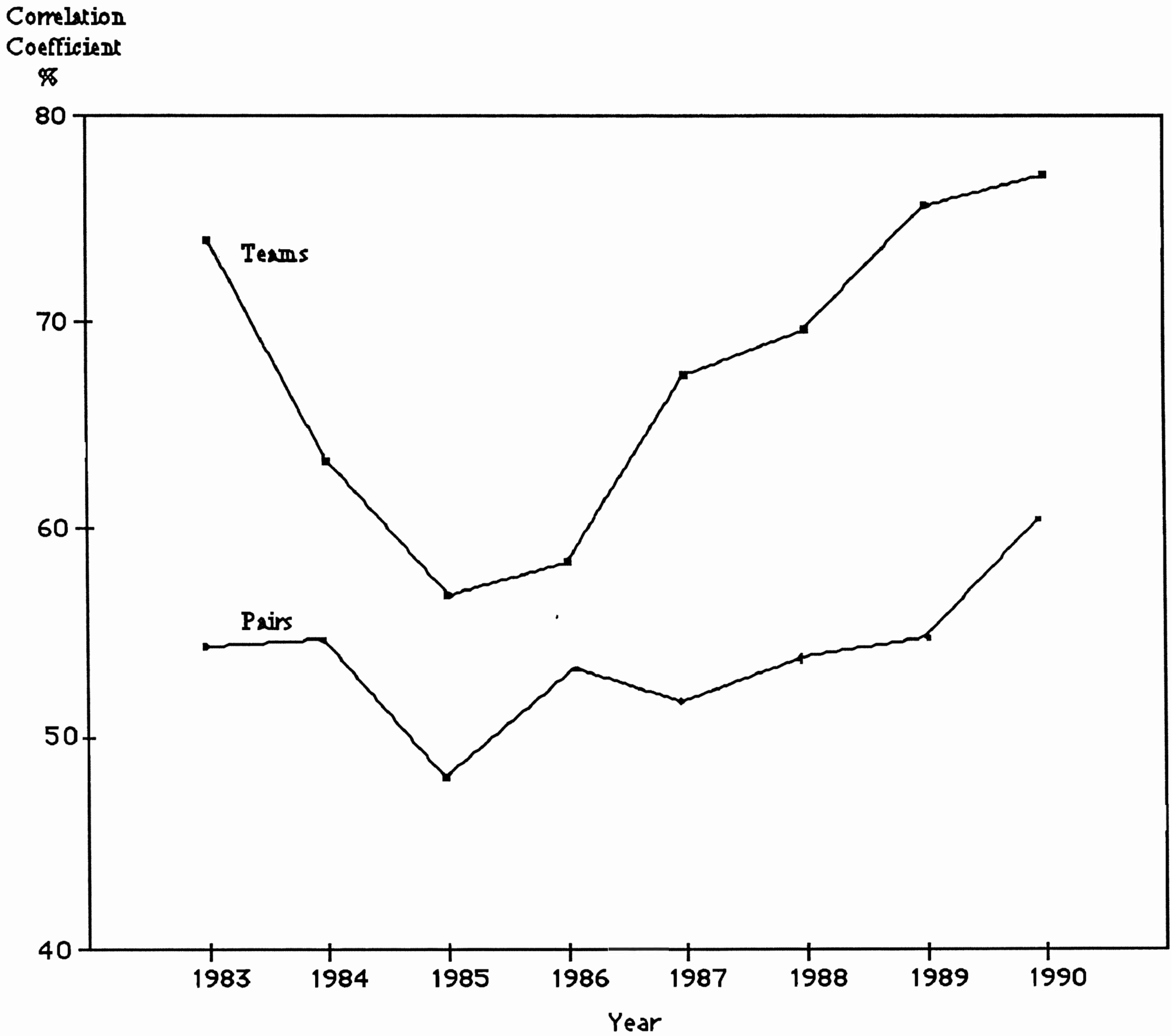

Figure 1

For each event, the result, along with the effect on ratings, is produced, as shown in Table 1. Also at periodic intervals ranked listings are produced.

Restricted Butler Pairs

Level of event : 1.10

Average input rating : 895.65 Average result : 0.00
Standard deviation : 169.52 Standard deviation : 78.33
Correlation : 0.7042

Name		Old rating	New rating	Event result	
Skinner	T	1082.57	1138.28	147.00	*
Thorp	B	1127.74	1183.45		
Lyons	Mrs A	850.00	845.58	-81.00	**
Dwyer	J F	945.25	940.83		
Hedley	A R	851.15	851.15	5.00	
Humphrey	L C	921.27	921.27		
	:		:		
	:		:		
	:		:		
Longmore	R C	1186.66	1186.66	68.00	
Price	Mrs J C	955.03	955.03		

No. of units : 28 No. varied : 9 Net variation : 7.69

Table 2

Conclusion

The experience with the duplicate bridge implementation has demonstrated the acceptability of a rating system based on team performances, both from the increased interest in the system compared to the original Masterpoint scheme, and from the system's demonstrated ability to better predict the relative strength of teams entering events compared to any other method, including subjective assessment by players and officials. This last characteristic has resulted in all seedings for events being related to rankings.

This is not to suggest that the system does not contain deficiencies. The most important deficiency involves the inability of the system to account for interactions between players within a team that are generally regarded as increasing the strength of a team. A second factor, not applicable in the bridge application, is the external environmental factor which may significantly affect the performance of one or more teams in a competition.

However within the framework of the system, these deficiencies can be eliminated, along with other factors specific to a particular sport, to provide the reliable ratings desired to enhance the activity.

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STRATEGY IN FELL RUNNING: AN ANALYSIS OF THE BOB GRAHAM ROUND

M Hayes[1] and J M Norman[1]

Abstract

The "Bob Graham Round" is an *arduous* route in the English Lake District which fell-runners attempt to complete in under 24 hours. Runners who complete the round and wish to have their achievement recognised must submit a detailed timetable of their progress. From an analysis of these submissions we have:

(a) tested an elaboration to the well-known Naismith rules for predicting progress in mountainous country,

(b) tested the conjecture that successful athletes in stamina events apply a constant work rate,

(c) estimated the effect on effort of such additional factors as darkness or fatigue.

The objective of our work is threefold. Firstly to help runners planning an attempt on the round. Secondly, (as the Lake District mountains are so well known and accessible) to provide any readers who are mountain walkers with an opportunity to check our estimates against their own experience and judgement. This can of course be done for any section of the route, and does not require the whole round to be tackled. Thirdly to make our findings applicable for mountainous areas other than the Lake District.

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1. Introduction

The Bob Graham Round itself consists of a traverse on foot of 42 specified peaks, starting and finishing at Keswick Moot Hall, to be completed in either a clockwise or anti-clockwise direction within 24 hours (see figure 1).

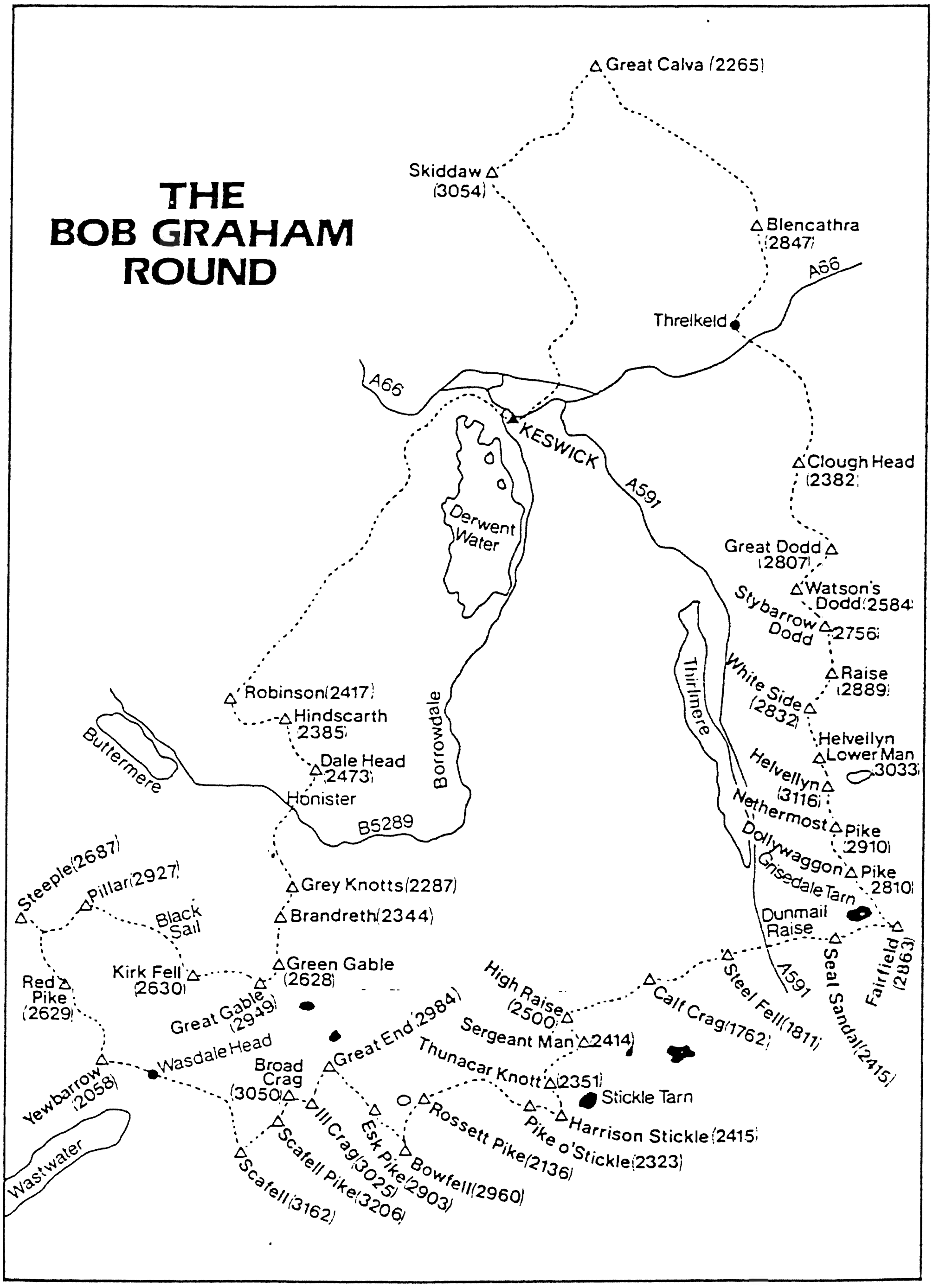

Figure 1. (from R. Smith (ed) "42 peaks: the story of the Bob Graham round")

Bob Graham's original round was completed in 1932 and his record stood for 28 years, but since 1960 more than 700 athletes have successfully completed the round. Every runner who successfully completes the round submits an application form for the ratification of a successful attempt on which he or she states the time at which each of the 42 summits was reached, together with the start and finish times for the circuit. Under the guidance of Mr Fred Rogerson, the archivist, data relating to many successful circuits of the Lake District peaks were collected from the archives of the Bob Graham 24 hour club. Although records were examined for circuits even more arduous than the Bob Graham Round, in this work the analysis was restricted to data for the round itself.

The data considered here relate to a sample of 56 successful attempts which are categorised below:

Anti-clockwise

Start times	Number	
0000-0059	4	
0100-0159	1	
0300-0359	2	
0700-0759	2	
0800-0859	13	(including one pair who ran together and one foursome)
0900-0959	9	
1000-1059	6	(3 pairs, one of which ran together)
	37	

Clockwise

Start times	Number	
0000-0059	5	(including one pair who ran together)
0100-0159	6	(including one threesome)
0200-0259	1	
0500-0559	2	
0800-0859	1	
1900-1959	3	(including one pair)
none given	1	
	19	

It was decided to work with three categories of runners: clockwise runners who left at midnight or soon after, anti-clockwise runners who left at midnight or soon after and anticlockwise runners who left in the morning between 0700 and 0859. Five runners were considered in each of the three categories.

Runners who were part of a group, whether or not the group ran together during the entire round, were not included in the analysis.

2. Measuring the work done in fell running

Many hikers have used the well-known Naismith rule (Mills [3]) to estimate the time taken to traverse a mountainous route: "Allow 20 minutes per mile + 30 minutes for every 1 000 feet climbed". The rule was extended by one of the authors (MH) when supervising and assisting fell runners to complete the Bob Graham Round to the following five rules:

(1) Flat or uphill. Allow 10 minutes per mile + 10 minutes per 1000 feet of ascent.

(2) Gentle downhill (under 500 feet per mile). Allow eight minutes per mile.

(3) Steep downhill (over 500 feet per mile). Allow 8 minutes per mile + two minutes per 1000 feet of descent.

(4) Rough ground on flat, uphill or gentle downhill sections. Allow 15 minutes per mile + 15 minutes per 1000 feet of ascent.

(5) Rough, steep downhill. Allow 10 minutes per mile + 10 minutes per 1000 feet descent.

"Rough ground" is defined as an area of crags requiring rock climbing skills to move about.

The revised rules are somewhat rough-and-ready, in distinguishing simply between ground which is rough and ground which is not, and between gentle and steep downhill slopes, but rules of this kind have to be simple to be usable. The revised rules have been shown to be useful in route choice and the siting of controls (Hayes and Norman [2]). If the ground-factors, as defined above, provide good estimates under favourable conditions, we can then extend our analysis by looking at excess times taken during unfavourable conditions and assigning their cause to fatigue, darkness or weather.

How do the times generated by our revised rules fit in with advanced schedules of others assembled from judgement and experience?

Many runners have a schedule to which they try to keep when running the round, and some runners enclose a copy of their schedule with their ratification form. In the sample there were three anti-clockwise schedules, one of which was used by four runners in three separate attempts. There were also four clockwise schedules, each for a separate attempt. There were only minor differences among the anti-clockwise schedules and the clockwise schedules, respectively. For further analysis, we took the anti-clockwise schedule used on three attempts in the sample, and a clockwise schedule which had been prepared for a foreign athlete by Mr Rogerson and others with an extensive knowledge of the circuit.

Our derived times were generated by a straightforward application of the revised Naismith rules to a schedule of distances with associated rises and falls, (Hayes [1]). Rough ground is noted and corresponds to the description of the route segment. Three sets of distances are given in the schedule: the first is from peak to peak as the crow flies, the second is derived from careful measurement of the routes on a $2\frac{1}{2}$ inch to the mile ordnance survey map, and the third takes account of a particular fell runner's intimate knowledge of all the extra twists and turns that cannot be shown by the $2\frac{1}{2}$ inch map. The total distance of the third set is 73.8 miles and the total height gained (and lost) is 25,900 feet. The distance corresponds well to the generally accepted figure of 72 miles. The height gained (or lost) is significantly less than the oft-quoted 27,000 feet, but our lesser figure is derived from a detailed analysis of the round when it is split into over 100 sections.

The total times for the clockwise and anti-clockwise routes predicted by the rules are 19 hours 21 minutes and 19 hours 34 minutes. The times are fast, and take no account of rests or of slower speeds in night-time running, (or of fatigue, illness or injuries which are more likely to affect later stages and which may influence people who design a schedule on judgemental grounds).

A total schedule is made up of 47 segments (between the start, the 42 peaks, 4 recognised resting points and the finish). We carried out a regression analysis for those segments comparing our derived times and other people's estimates.

The agreement between our derived times and other people's advanced schedules for both clockwise and anti-clockwise schedules is quite good (R^2 = .89 and R^2 = .84 respectively with the schedule times averaging 5% and 2% more than the derived times).

A linear regression was also carried out of the segment times for J S Bland's record run of less than 15 hours, which was completed entirely in day light. J S Bland's times were on average 20% less than the derived times, with an R2 value of .94. One would expect a high correlation between our derived times and this record-holder as he would not have had to cope with darkness, nor would fatigue be such a problem for him.

The plots of residuals were satisfactory, except that the derived times for the initial and final segments (in large part on roads from and to Keswick) were under-estimated by the revised rules. In general, it was considered that the revised rules could be used as an indication of the time taken to traverse a given route in mountainous country at a constant level of effort, and hence as a measure of work done.

3. Measuring the pace (work rate) of fell runners

For the purpose of this paper, the peaks have been numbered anti-clockwise. Each runner's total elapsed time was divided into eight equal parts and interpolation was used to estimate the position reached by the runners when they had been running for one-eighth, one quarter, three-eighths, of their total elapsed time. Thus a clockwise runner who completed the round in exactly 24 hours who was at Bowfell (peak no 19) 11 hours 45 minutes after his start and at Esk Pike (peak no 20) 12 hours 10 minutes after his start would be reckoned as being three-fifths of the way between them (position 19.6) halfway through his round. The positions reached by each of the 15 runners calculated in this way are given in the following table:

Anti-clockwise midnight starts

AN1	AN2	AN3	AN4	AN5
K	K	K	K	K
3.5	3.9	3.9	3.8	3.6
9.1	10.4	11.7	9.6	10.1
12.9	13.3	16.9	13.0	13.1
20.3	21.3	25.6	21.0	20.5
27.8	28.3	29.7	27.9	27.5
38.1	38.1	39.2	37.5	n.a.*
40.7	40.7	41.0	40.3	40.2
K	K	K	K	K

(* the runner's watch stopped)

Anti-clockwise morning starts

AM1	AM2	AM3	AM4	AM5
K	K	K	K	K
6.2	5.5	3.9	5.7	4.3
12.2	12.0	11.6	12.1	11.7
19.1	17.7	14.3	16.2	16.8
27.0	27.1	23.6	25.5	25.6
31.4	30.3	28.7	28.8	30.0
39.2	38.2	38.2	36.7	39.0
40.8	40.5	40.2	40.1	40.5
K	K	K	K	K

Clockwise midnight starts

CN1	CN2	CN3	CN4	CN5
K	K	K	K	K
40.3	40.4	40.5	40.3	40.6
37.1	35.5	36.6	35.0	36.1
27.5	27.0	27.7	27.0	27.3
19.5	19.2	20.3	19.6	19.6
12.6	12.5	12.9	12.7	12.6
8.7	8.7	9.5	9.2	n.a.
3.1	3.2	3.6	3.7	n.a.
K	K	K	K	K

The results are shown diagrammatically in Figure 2, which represents the Bob Graham circuit with the changes in direction ironed out and with only the distances between the peaks, as the crow flies, preserved.

From the schedules derived using the revised Naismith rules we drew up the following position tables in the manner already described.

Anti-clockwise	Clockwise
K	K
2.7	40.7
8.6	38.3
12.6	27.9
19.7	20.3
27.4	12.9
35.6	9.3
40.3	3.7
K	K

These positions are marked on Figure 2, for easy comparison with the positions reached by the fifteen runners in the sample.

The schedules could be considered as times taken to achieve the 43 segments of the complete circuit (or task): times to get from Keswick to peak 1 from peak 1 to peak 2, and so on (neglecting rest points). Thus on the anti-clockwise schedule, one-eighth of the work (task) has been done by the time the runner has reached position 2.7, one quarter by position 8.6, and so on.

In a related way, we can compute the "work done" by each of the fifteen runners by the time they have used one eighth of their elapsed time, one quarter, and so on. For example, runner AN 1 reached position 3.5 at one eighth of his elapsed time, but in the 19 hours 34 minutes anti-clockwise schedule, position 3 and position 4 are scheduled at 2 hours 23 minutes and 3 hours 1 minute after the start. Interpolating, position 3.5 is scheduled for 2 hours 42 minutes after the start. 2 hours 42 minutes divided by 19 hours 34 minutes is .15, so that 15% of the work has been done in 12.5% of the runner's elapsed time.

Calculations carried out in this way for all 15 runners are shown below:

Anti-clockwise midnight starts

AN1	AN2	AN3	AN4	AN5
.15	.16	.16	.16	.15
.12	.13	.16	.12	.14
.13	.12	.15	.13	.12
.12	.13	.12	.13	.11
.12	.12	.10	.10	.11
.13	.11	.11	.12	
.12	.12	.11	.11	.24
.11	.11	.09	.13	.13

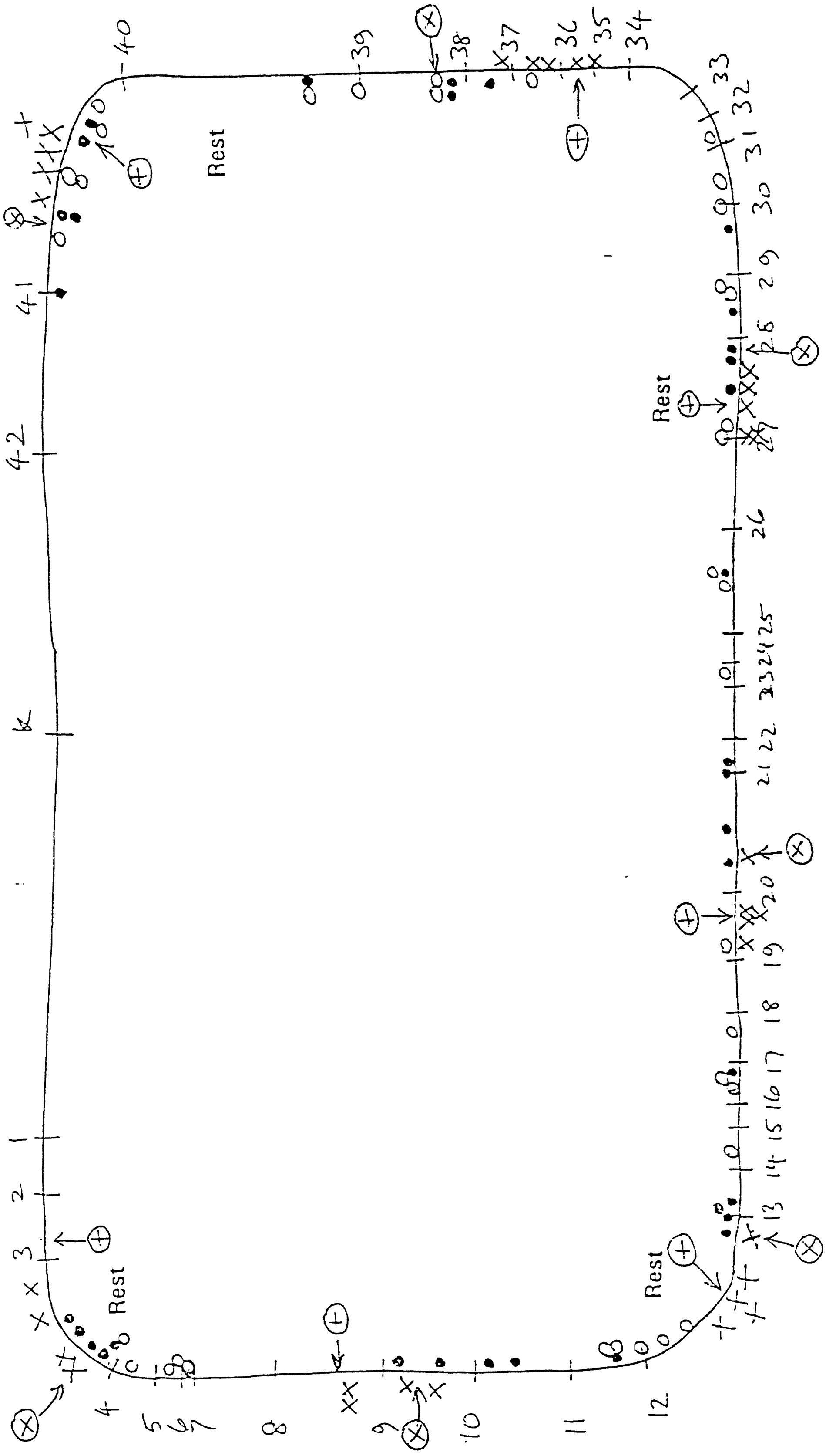

Figure 2. Position reached by various runners.

•	Anti-clockwise midnight starts	+	A-C schedule
o	Anti-clockwise morning starts	X	C schedule
X	Clockwise midnight starts		

Anti-clockwise morning starts

AM1	AM2	AM3	AM4	AM5
.18	.18	.16	.18	.16
.17	.15	.16	.18	.16
.14	.14	.11	.09	.13
.12	.15	.14	.14	.16
.09	.08	.09	.08	.09
.10	.07	.11	.09	.09
.10	.11	.10	.10	.09
.10	.12	.13	.14	.12

Clockwise midnight starts

CN1	CN2	CN3	CN4	CN5
.15	.14	.14	.15	.13
.11	.14	.13	.14	.14
.13	.14	.11	.12	.13
.13	.11	.12	.11	.12
.12	.13	.13	.12	.13
.13	.11	.12	.11	.11
.12	.11	.12	.12	.12
.11	.12	.13	.13	.12

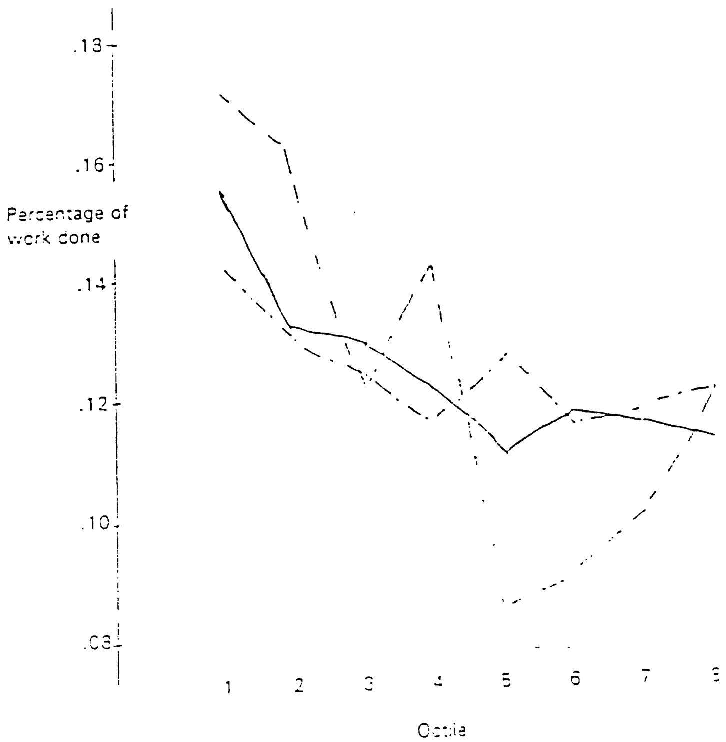

Figure 3. Percentage of "work done" in each octile of the round by three categories of runner.

_ _ _ morning start, anti-clockwise

______ midnight start, anti-clockwise

_ • _ • _ • midnight start, clockwise

The average "percentage work done" in each eighth of their elapsed time is plotted for the three categories of runner in Figure 3. When the reduced pace of the morning start runners in their night section is considered (AM 1-5 in octiles 5 and 6) then in broad terms, all the runners tend to start fast and go slower, although in the second half the pace is fairly even.

4. The effect of darkness on pace (work rate)

Inspection of Figure 3 suggests a reduction in pace of about 20%, and this was confirmed by a more detailed analysis. Some segments of the anti-clockwise course were run in darkness by the morning start runners and in day time by the midnight start runners, and vice versa.

The method of estimation was as follows:

(1) For each runner, the total anti-clockwise scheduled time (19 hours 34 minutes) was divided by the runner's total time to obtain a rating (a measure of the overall pace or work rate of each runner).

(2) For each of four segments, the scheduled time was divided by the time taken by each runner to obtain a work rate for that segment for each runner.

(3) (2) was divided by (1) to obtain a relative work rate for each runner for each segment.

(4) The relative work rates for all runners in a category were averaged to obtain an average relative work rate for each segment.

(5) The average relative work rate for each segment by day time runners was compared with that by night time runners to obtain an estimate of the proportionate loss of pace due to darkness.

The calculations are given below:

	Segment work rates				Overall work rate	Relative work rates			
	Keswick Honister	Honister Green Gable	Raise Threlkeld	Blencathra Great Calva		K-H	H-GG	R-T	B-GC
AN1	.91	.70	1.01	.93	.81	1.12	.86	1.25	1.15
AN2	1.13	.83	.76	.82	.83	1.36	1.00	.92	.99
AN3	1.07	.90	.67	.75	.78	1.37	1.15	.86	.96
AN4	1.06	.77	.80	.77	.80	1.33	.96	1.00	.96
AN5	1.05	.88	n.a.	.84	.84	1.25	1.05	n.a.	1.00
Average						1.29	1.00	1.01	1.01
AM1	1.29	1.07	.56	.52	.79	1.63	1.35	.71	.66
AM2	1.19	1.05	.49	.76	.79	1.51	1.33	.62	.96
AM3	1.19	.98	.68	.83	.86	1.38	1.14	.79	.97
AM4	1.27	.92	.63	.98	.80	1.59	1.15	.79	1.23
AM5	1.27	1.02	.62	.71	.89	1.43	1.15	.70	.80
Average						1.51	1.22	.72	.92

The estimated percentage reduction in work done in the four segments owing to night time running is:

Segment	Percentage reduction
Keswick - Honister	15
Honister - Green Gable	18
Raise - Threlkeld	29
Blencathra - Great Calva	9

5. Discussion

It is an obvious point that running up and down hills in the dark takes significantly longer than it does in the day time. For this reason, most attempts at the Bob Graham round take place in midsummer. Thus, a midnight start, which limits night time hill running, seems better than a day time start.

The advantage of the clockwise circuit over the anti-clockwise circuit is not entirely clear, but it could be argued that a midnight start clockwise circuit reduces the number of night time summits to a handful (peaks 42, 41, 40 going outwards, and possibly none on the return), whereas on an anti-clockwise circuit about ten summits may have to be climbed in the dark. All this, of course, relates to the majority of successful runners who complete the round in 21- 24 hours, and not to extraordinary athletes such as J S Bland, who was able to complete the circuit in under 15 hours, entirely in daylight.

Of the runners in the sample who left at midnight or soon after, the majority made the clockwise circuit.

Many rounds are begun in the morning rather than at night, but this may be simply because of the difficulty of getting support teams in place in time for a night time start. Few contestants can mobilise their support for a midnight start on a Friday, and a Saturday midnight start would not give much time for rest and recovery before the start of the next working week on Monday morning. As one successful runner told us, "A morning start allows a prior night's sleep. Conversely, a midnight start means you have probably just finished a day's work!" The optimal start-time may not be feasible for a runner constrained by the demands of the working week.

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APPLYING LEAST SQUARES TO TEAM SPORTS AND OLYMPIC WINNING PERFORMANCES

Raymond Stefani[1]

Abstract

Least squares is applied to a model for the margin of victory in team sports to estimate the home advantage and to estimate ratings that can be used for prediction. The home advantage is compared for seven types of competition. For prediction, least squares rating differences must be reduced using a shrinking factor. Predictions are applied to five kinds of team competition including over 12,000 games. Least squares is also used to estimate the rate of improvement of Olympic winning performances. A comparison is made of the rate of improvement for different types of events and of the rate of improvement in athletics for all Olympic games since 1896 showing the influence of international events. The percent difference is examined between the winning men and women in swimming, running and speed skating as a function of distance.

1.0 Introduction

Least squares (LS) methods have been used in such diverse applications as modelling the performance of aircraft to aid in stabilization and estimating trends in the economy so as to aid in investments, pricing and personnel management. In comparative studies by Stefani [3], Stefani and

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Clarke [10] of LS versus other statistical methods of prediction, no other method significantly outscored the LS method regarding selecting the winning team and estimating the actual margin of victory. LS methods provide a simple and effective mathematical tool of general applicability.

In this paper, LS is applied to team sports and to Olympic winning performances to provide insight into athletic performance, summarizing over twenty years of research [1-9]. This paper is organized as follows. In Section 2, the margin of victory in team sports is analyzed. LS estimates are made of the home advantage and the ratings of the teams. Home advantage is examined for seven types of competition. Predictions are formed by applying a shrinking factor to the ratings and including the home advantage. Predictive results are summarized for five kinds of team competition including over 12,000 games. In Section 3, LS is applied to Olympic winning performances and some interesting results appear regarding differential improvements by types of event, different achievement levels of men and women as a function of distance, and the effect of international events upon the rate of improvement. Section 4 contains the conclusions.

2. Team Sports

A variety of schemes exist to rank teams and individuals and thereby to predict the outcome of a subsequent competition and to seed competitors in a tournament. These schemes are either accumulative or adjustive. An accumulative scheme results in the accumulation of points and rankings based on that total. The point total never diminishes and it can be limited to some maximum value. Most soccer tables of standings are ranked with two points accruing for a win and one point for each draw. The World Cup of Skiing uses an accumulative system with provisions for limiting total points.

An adjustive scheme causes ratings to rise or fall as performance is above or below some predicted level based on the previous ratings. For example, the World Chess Federation uses the Elo system in which the rating difference between each competitor and the average opponent provides an estimate for the number of victories for each competitor. The competitor's rating changes as a function of the actual number of victories compared to that target.

Adjustive schemes are also used to rate teams in a variety of sports and then to predict the outcome of the next competition. A few schemes use relatively large quantities of offensive and defensive statistics, although most schemes operate only on margin of victory adjusted for home field advantage.

The least squares (LS) method is a very useful adjustive scheme since the method is computationally efficient and no learning of coefficients is needed. To facilitate applying LS to team sports, the index i represents some reference team, m represents the week of the season and j(m) represents the opponent of team i during week m. The independent variables are i and m since j is dependent on the schedule and is completely determined by i and m. Primary interest focuses on data for the win margin, the number of points scored by a team minus those scored by the opponent and on the home field advantage. A model for win margin is

$$w_{ij(m)} = h_{ij(m)} + r_i^m - r_{j(m)}^m + e_{ij(m)} \tag{1}$$

where $w_{ij(m)}$ represents the win margin for team i against opponent j in week m, $h_{ij(m)}$ is the home advantage for team i, r_i^m is the rating for team i using data including week m, $r_{j(m)}^m$ is the similar rating for the opponent and $e_{ij(m)}$ is a zero-mean random error.

The home advantage and ratings can be found to minimize the sum of the squared errors from (1) where there are N teams and K weeks have been completed

$$J = \sum_{i=1}^{N} \sum_{m=1}^{K} e_{ij(m)}^2 \tag{2}$$

There are three possible interpretations of home advantage; that is, $h_{ij(m)}$ can be interpreted as a single value for all teams, as a distinct value for each team or as a distinct value for each combinations of teams. The second and third interpretations require the use of decreasing numbers of games with an increasingly large error variance; hence, attention is focused on the first interpretation which uses all games played. If a single value is to be found, then $h_{ij(m)}$ is interpreted as +h if i plays at home, as -

h if i plays away or as 0 if i and j play on a neutral ground or on a ground that both teams use as a home ground. It follows that the LS value of h minimizing (2) for M home games is

$$h = (1/M)\left[\sum_{\substack{i=1 \\ i \text{ at home}}}^{N}\sum_{m=1}^{K} w_{ij(m)} + \sum_{\substack{i=1 \\ i \text{ at home}}}^{N}\sum_{m=1}^{K} (r^{m}_{j(m)} - r^{m}_{i})\right] \tag{3}$$

The right-most summation of (3) is the total rating difference between the visitors and the home teams. Over a large number of games, the average team plays at home as often as away; hence,the average team appears as $r^{m}_{j(m)}$ half the time and as r^{m}_{i} half the time, causing right-most summation to be nearly zero so that (3) simplifies to

$$h = (1/M) \sum_{\substack{i=1 \\ i \text{ at home}}}^{N}\sum_{m=1}^{K} w_{ij(m)} \tag{4}$$

which is the average margin of victory for the home team, found off-line from the ratings.

Table 1 contains data for seven kinds of team competition. The data includes the number of games, the fraction of games won by the home team, the fraction of draws, the home-win-minus-away-win fraction HW-AW, the home advantage h expressed in score units (goals, points, etc.) per game, the total score per game and the total-score-to-home-advantage ratio R which decreases as HW-AW increases.

I have collected data for the three Europe Cups (Champions Cup, Cup Winners Cup and EUFA Cup) during seven seasons (1981-82 and 1985-86 through 1990-91), soccer data for six nations during two seasons (1980-81 and 1981-82), data for USA college and professional football during four seasons (1979-80 through 1982-83) and data for major league USA professional baseball during the 1982 American League and National League seasons. Data for the National Hockey League (USA/Canada) was provided by Cleroux (Univ. Of Montreal) for four seasons (1975-76 though 1978-79) and data for Australian rules football (Australian Football League) was provided by Clarke (Swinburne Inst. of Technology) for ten seasons (1980 through 1989).

Table 1. Home Advantage

Sport	Games	Home Win (HW)	Draw	(HW-AW)	Home Advantage (h)	Total Points (T)	R (T/h)
Soccer 3 Europe Cups	1356	0.580	0.209	0.369	0.91	2.70	3
Soccer * 6 Nations	6601	0.485	0.281	0.251	0.45	2.72	6
Hockey (USA)	2840	0.505	0.166	0.176	0.68	6.76	10
College Football (USA)	1669	0.574	0.017	0.165	3.71	43.0	12
Prof. Football (USA)	671	0.574	0.003	0.151	3.27	40.7	12
Australian Rules Football	1109	0.580	0.007	0.167	9.8	206.5	21
Baseball (USA)	2106	0.538	0.000	0.076	0.26	8.9	34

* England, Germany, Italy, Norway, Spain, Switzerland

To interpret data such as that in Table 1, authors such as Pollard [11] suggest that the home advantage, perhaps more properly the visiting disadvantage, is due to physiological factors which are detrimental to the visiting team such as travel fatigue, psychological factors such as crowd intimidation of the visiting team and tactical factors such as lack of familiarity of the visiting team with the playing surface. Each competition on Table 1 has a specific absolute and relative mix of the three factors, causing home advantage to rise or fall accordingly. Certainly when one club visits an opponent in another nation for Europe Cup competition, all those factors are strongly in evidence. The Europe Cup competition features home-away pairs; hence, it is significant that one goal in three is attributable to home advantage and that the home team wins 36.9% more often than it loses. Soccer competition entirely within a given nation results in a lower home advantage (one goal in six). Hockey, USA college football and USA professional football have similar home advantages (about one point in 12). A lower home advantage (one point in 21) occurs in Australian rules football where most games are played in the greater

Melbourne area. However, expansion of the game into other parts of Australia has resulted in a predictably large advantage for the West Coast Eagles [9]. Finally, USA professional baseball teams tend to play up to four games at each location so that most home advantage factors are minimized and the value of HW-AW is only 7.6% with only one run in 34 being due to the home advantage.

Now that h is available from (4), the LS rating for team i which maximises (2) using K weeks of data is

$$r_i^K = \frac{1}{n(i)} \left\{ \sum_{m=1}^{K} [wa_{ij(m)} + r_{j(m)}^m] \right\} \tag{5}$$

where team i has played n(i) games during the K weeks of the season over which the ratings are computed and where $wa_{ij(m)}$ is the adjusted win margin

$$wa_{ij(m)} = w_{ij(m)} - h_{ij(m)} \tag{6}$$

The recursive equivalent for (5) is

$$r_i^K = \frac{1}{n(i)} \left\{ [n(i) - 1]\, r_i^{K-1} + wa_{ij(K)} + r_{j(K)}^K \right\} \tag{7}$$

A similar equation can be written for team j by reversing the indices i and j. There are two unknowns in the two equations, the ratings r_i^K and r_j^K. These equations can be solved simultaneously to yield the LS recursive algorithm for r_i^K.

$$r_i^K = r_i^{K-1} + \frac{n(j) - 1}{n(i)n(j) - 1} \left\{ wa_{ij(K)} - [r_i^{K-1} - r_{j(K)}^{K-1}] \right\} \tag{8}$$

Adjustments can be limited to two standard deviations to reduce large rating changes due to anomalous results.

Attention now turns to predictions. Future margins of victory tend to exceed the differences between LS ratings, so that a model for the margin of victory in the next game (to be played during week K+1) is

$$w_{ij(K+1)} = h_{ij(K+1)} + L\left[r_i^K - r_{j(K+1)}^K \right] + e_{ij(K+1)} \qquad (9)$$

where L is a shrinking factor between zero and one which reduces each subsequent prediction compared to the rating difference. The sum of squared prediction errors in (9) can be minimized by selecting L based on past predictions. Compactly, L is

$$L = \frac{\text{sample covariance } [wa_{ij(K+1)} \, (r_i^K - r_{j(K+1)}^K)\,]}{\text{sample variance } [r_i^K - r_{j(K+1)}^K\,]} \qquad (10)$$

The predicted win margin for the next game becomes

$$\hat{w}_{ij(K+1)} = h_{ij(K+1)} + L\,[r_i^K - r_{j\,(K+1)}^K\,] \qquad (11)$$

Table 2. Predictions

Sport	Seasons	Games	L	Correct Prediction	Average Absolute Error
College Foot. (USA, 4 year)	11	6137	0.87	0.718	13.9
College Football (USA, 2 year)	3	536	1.00	0.700	12.5
Prof. Football (USA)	11	2148	0.66	0.657	11.7
USA College Basketball	2	1926	1.00	0.690	9.8
Australian Rules Football	10	144	0.66	0.681	32.9
Total		12193		0.698	

In summary, LS ratings are computed by (8) using previously calculated h then (11) is used for prediction using previously calculated L.

Table 2 summarizes predictions of five types of team competition using the LS method. Table 2 contains the number of seasons, games played, L, the fraction of games in which the predicted team actually won (draws were considered half right and half wrong) and the average absolute error between the true and predicted margin of victory. USA four-year college and professional football predictions spanned the 1970-71 through 1980-81 seasons. USA two-year college football predictions covered the 1977 through 1979 seasons. USA college basketball predictions covered the 1972-73 and 1973-74 seasons. Australian rules football is for the 1980 through 1989 seasons. Values of L other than one were not attempted for USA college basketball and for USA two-year college football. Except for Australian rules football, the predictions were made before the fact and most of those predictions were published in newspapers. The first eight years of Australian rules football predictions were applied to old data where the computer using the LS method could not look ahead; hence, the predictions emulated real-time accuracy. The last two seasons of Australian rules football predictions were made in real time.

For 12,193 games the correct winner was selected 69.8% of the time. In a two-year contest covering 638 USA football games, the computer outperformed the average newspaper predictor by 2.5% at selecting the correct winner. In a similar two year study of 276 Australian rules football games, the computer outperformed the average newspaper selector by 3.4%. However, the computer is not quite as good as a highly skilled predictor, such as those who produce the selections that gamblers in the USA must pick against. In a study of 2435 games [3], the booking agency's selector outperformed the computer by 1% at picking the winner and by 2.2% at predicting the actual margin of victory. The 2.2% difference represented only 0.27 points per game out of an average absolute error of 11.9 points per game. Considering that over 40 points are scored per game, little actual difference existed between the computer predictions and the highly skilled predictor.

3. Olympic Winning Performances

The winning performances in about one-quarter of the approximately 250 events in the summer Olympics provide meaningful comparisons in that the competitive conditions are reasonably similar. Athletics (called track and field in some nations), swimming and weightlifting provide a convenient pool of data from which some interesting trends may be derived. Conversely, sports such as yachting with variable wind conditions and gymnastics with subjective judging are inappropriate for purposes of seeking trends.

Where trends are sought, the previous winning performances may be modelled by a first order autoregressive process

$$w_m = a\, w_{m-1} + e_m \tag{12}$$

where w_m and w_{m-1} are the winning performances in consecutive Olympics with indices m and m-1 respectively, e_m is a zero-mean random error and a is a regression coefficient that can be selected as the LS value minimizing

$$J = \sum_{m=2}^{K} e_m^2 \tag{13}$$

where there are K successive past results. Thus, for each event

$$a = \sum_{m=2}^{K} w_m w_{m-1} \Big/ \sum_{m=2}^{K} w_{m-1}^2 \tag{14}$$

If there is improvement in a timed event, a is less than one, while for improvement in measured events such as jumping, discus and weightlifting, a is greater than one. I have developed a more convenient measure of improvement, the percent improvement per Olympiad, %I/O

$$\%I/O = \begin{cases} 100\,(a-1) \text{ for measured events} \\ 100\,(1-a) \text{ for timed events} \end{cases} \tag{15}$$

If a is 0.99 for a timed event and 1.01 for a measured event, both events would have a %I/O of 1%.

In general, a rate of improvement is due to some combination of four factors: first, the health and size of the pool of athletes from whom the Olympic teams are formed; second, the techniques available to the athletes; third, the effectiveness of the coaches at training the athletes in fitness and technique; and fourth, the conditions under which the competition is held. A number of interesting trends may be evaluated by calculating long-term %I/O over several successive Olympics and short-term %I/O using only two successive Olympic results. Figure 1 shows the result of calculating the %I/O for each of 45 events using ten successive Olympic results spanning 1952-1988. The values of %I/O were then averaged for running, jumping, swimming, throwing and weightlifting events for men and women, except for weightlifting which had no women's competition. Two conclusions are evident: the rate of improvement for women exceeds that of men for each type of event and the rate of improvement follows in rank order of the amount of leverage (mechanical advantage) applied, since running, jumping, swimming, throwing and weightlifting require roughly increasing amounts of leverage, and since those events exhibit increasing value of %I/O.

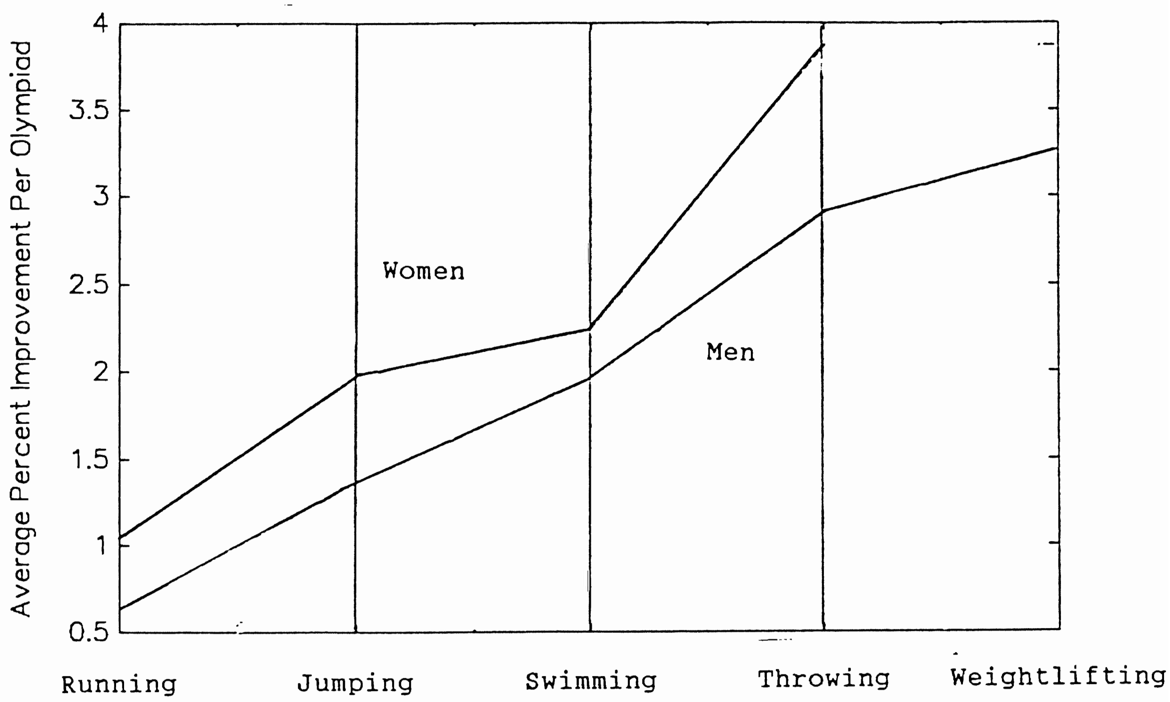

Figure 1. Percent Improvement per Olympiad 1952-1988 Average by Type of Event

The effect of international events upon rates of improvement may be seen by taking the value of %1/O for each men's athletics event over each pair of successive Olympics and averaging across all the men's athletics events (eleven events in 1900 increasing to 19 in 1920). The result is Figure 2. For more details see [8]. The greatest average %I/O was 11% in 1900, probably due to the increase in the number of athletes by at least a factor of four. The average %I/0 has remained under 3% since. The pattern around the time of WW1 (1912, 1920 and 1924) was repeated around the time of

WW2 (1936, 1948 and 1952). There was a build up in nationalism culminating in a large pre-war rate of improvement in 1912 and 1936. Just after each war there was negative (1920) or negligible (1948) improvement. The second Olympics after each war (1924, the "Chariots of Fire" Olympics and 1952) resulted in substantive rates of improvement similar to pre-war highs. The period from 1952 through 1968 was marked by spirited competition between the USA and the USSR and by high rates of improvement. Conversely, rates of improvement were lower during the boycott by African nations in 1976, western-bloc nations in 1980 and eastern-bloc nations in 1984. The Seoul Olympics of 1988 marked the return of all major nations to competition with a resulting increase in the rate of improvement.

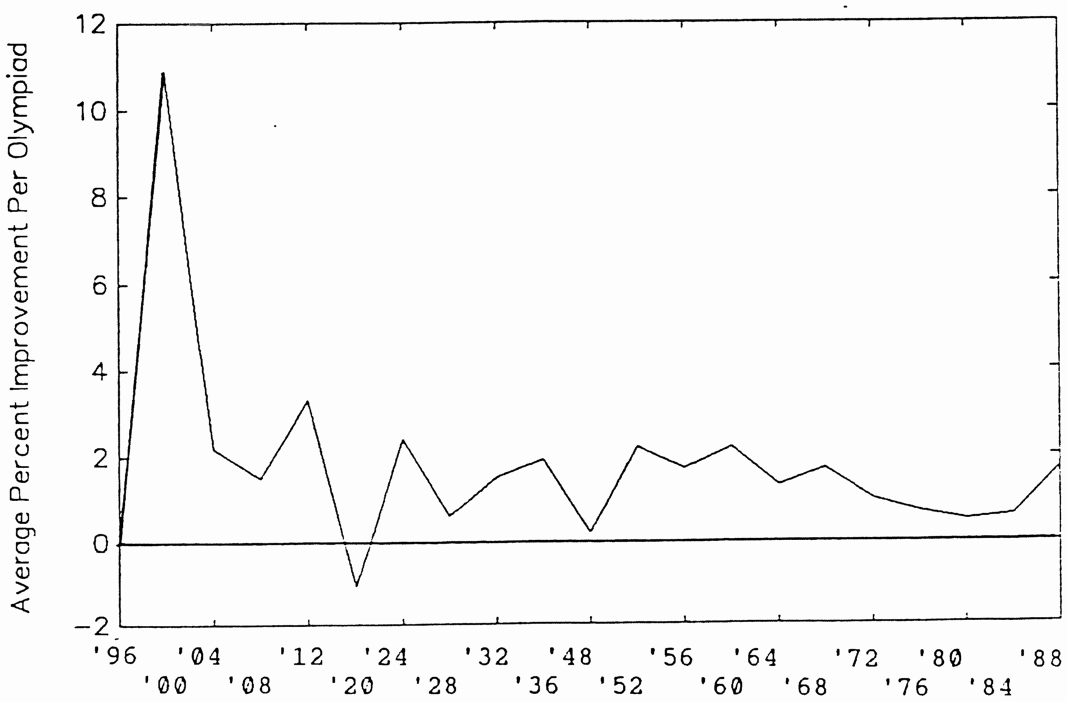

Figure 2. Percent Improvement for Each Olympiad Averaged for Men's Athletics

A prediction may be made for each subsequent Olympic event using

$$\hat{w}_{K+1} = a\, w_K \tag{16}$$

I have published predictions for 46-68 events per Olympics in athletics, swimming and weightlifting prior to the 1976-1992 summer Olympics [4-8]. For those prior predictions, the average absolute error was 2% in athletics and swimming and 7% in weightlifting.

An individual performance can be ranked based on the percent error between the actual performance and the prediction; that is, by the amount that a performance exceeds prior trends in improvement. Table 3 covers 46-68 events per Olympics (387 total events) in weightlifting, swimming and athletics from 1964-1988. While many people believe that Bob Beaman's 1968 long jump was the best ever, Table 3 suggests that Faina Melnik (SOV) and Ian O'Brian (AUS) had somewhat better winning performances among the athletics and swimming events. The most prodigious winning performance was that of Naim Suleymanoglu of Turkey (60 kg weightlifting) who lifted more than three times his body weight in the jerk in 1988. He broke the world record in the snatch, jerk and combined. In fact, his two-lift total was more than Paul Anderson lifted in 1956 to win the unlimited weight gold medal. Anderson weighed 138 kg (303 pounds) while Suleymanoglu only weighed 60 kg (132 pounds). It is fitting that Suleymanoglu is called the "Pocket Hercules".

Table 3. Superlative Individual Performances (1964-1988) for Weightlifting (7-9 Events), Athletics (28-33 Events) and Swimming (11-26 Events)

Event	Olympic	Winner	Performance	% Above Expectation
Weightlifting				
60 kg	1988	Naim Suleymanoglu (TUR)	342.5 kg	15.9
110 kg	1988	Yuri Zakharevich (SOV)	455.0 kg	13.2
52 kg	1988	Sevedalin Marinov (BUL)	270.00 kg	11.9
Athletics and Swimming				
Wom. Discus	1972	Faina Melnik (SOV)	66.62 m	7.7
Men 200 Br.	1964	Ian O'Brian (AUS)	2.27.80	7.5
Men Long Jump	1968	Bob Beaman (USA)	8.90 m	7.4
Wom. Discus	1988	Martina Hellmann (GDR)	72.30 m	7.2
Men Pole V.	1964	Fred Hansen	5.10 m	6.3
Men Javelin	1968	Janis Lusis (SOV)	90.10 m	5.3

An LS estimate of the percent difference between the winning woman and man in a common event can be found by averaging over successive Olympics. Figure 3 displays swimming data averaged for the five Olympics from 1976 through 1988 where the distance scale is repeated to better display data for each stroke. In general, the percent difference drops with distance except for the breaststroke in which the average difference is about the same at each distance. It may be that absolute differences in performance are due to differences in absolute muscle mass, while the greater buoyancy of women (due to a greater percentage of body fat) conveys a marginal advantage (less disadvantage) at longer distances as does certain differences in muscle types.

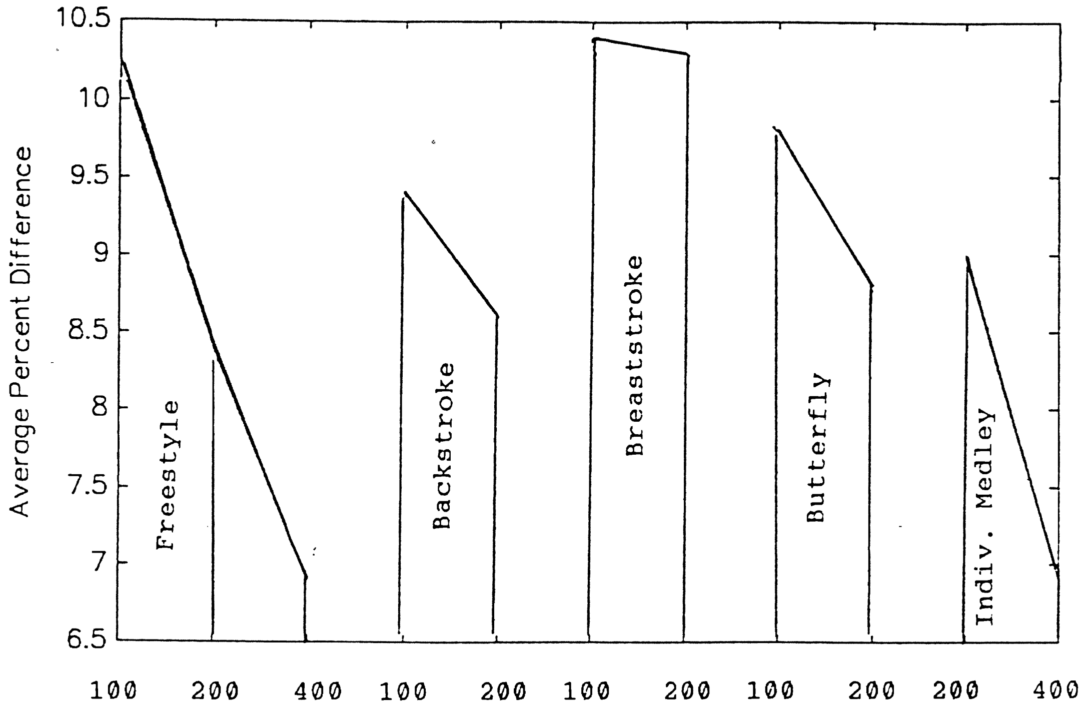

Figure 3. Average Percent Differences between Winning Men and Women Swimmers (1976-1988) Versus Metric Distance by Stroke

Figure 4 shows the percent difference versus distance in athletics averaged from 1976 through 1988 and a similar treatment of speed skating from 1976 through 1992. In athletics, the percent difference rises from 100 m through 10 km and then drops off for the marathon. In speed skating, the percent difference rises from 500 m through 1500 m and then drops off at 5000 m. The percent differences for each of the first three distances in athletics and speed skating are nearly identical. It may be that absolute differences in performance are due to differences in absolute muscle mass, while differences in muscle types convey some additional disadvantages to women at shorter distances and some marginal advantages to women at longer distances. A more definitive physiological analysis in needed to fully explain Figures 3 and 4.

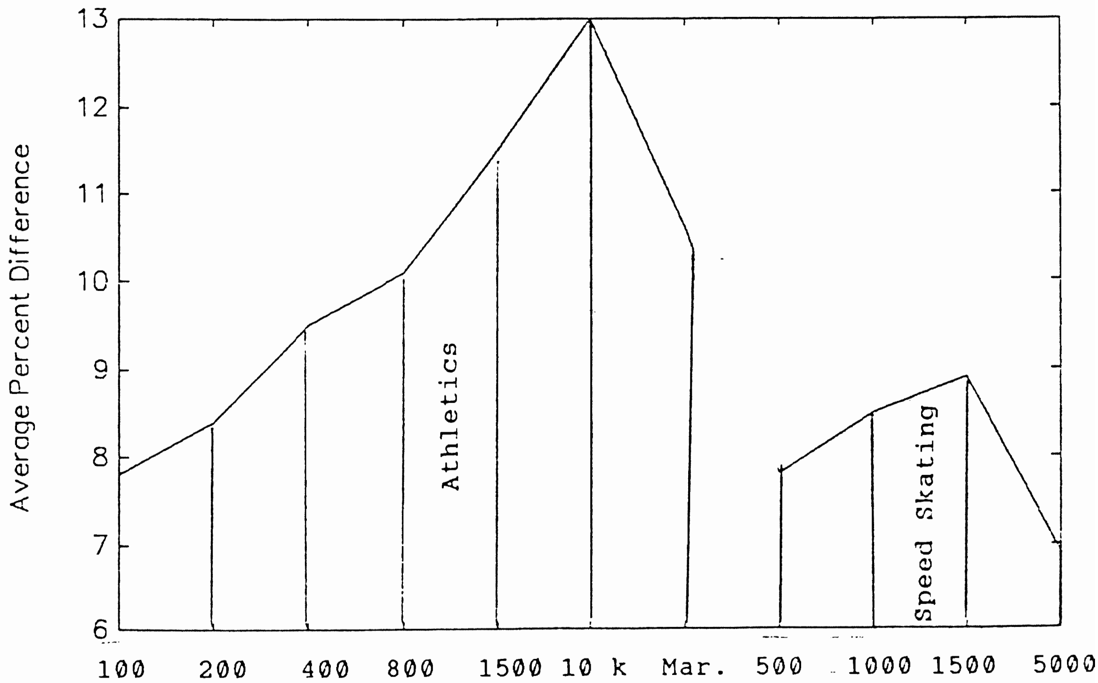

Figure 4. Average Percent Differences Between Winning Men and Women Runners (1976-1988) and Speed Skaters (1976-1992) Versus Distance

4. Conclusions

The least squares method may be applied to a model for the margin of victory in team sports. The resulting home advantage was studied for seven types of sports competition. The greatest home advantage is for

European Cup soccer competition among clubs. In that competition the factors that influence home advantage are most acute: psychological factors such as crowd intimidation, physiological factors such as travel fatigue and tactical factors such as lack of familiarity with the playing surface, all of which differentially affect the visiting team in a negative way. The types of competition with lower home advantages tend to have lower incidences of these factors.

The home advantage may be removed from winning margin data and ratings for each team may be found. These ratings along with a shrinking factor and the home advantage may be used for prediction. LS methods were applied to five kinds of sports competition using over 12,000 games with the correct winning team being selected 70% of the time. The LS predictor is superior to the average human selector and is slightly worse than a highly skilled selector such as those employed by gambling agencies to provide a benchmark to gamble against.

Olympic winning performances may be modelled as a first order auto-regressive process. The resulting percent improvement per Olympiad (%I/O) provides a number of interesting case studies. Women are improving faster than men across the spectrum of events. The greatest rate of improvement coincides with the greatest amount of leverage; that is, weightlifting, throwing, swimming, jumping and running have decreasing values of %I/O. International events have a strong influence on the %I/O. The periods around WWl and WW2 have similar patterns: more improvement due to nationalistic fervor prior to the wars, a sharp drop in improvement just after each war followed by a sharp increase in improvement as normalcy is restored. The period from 1952-1968 exhibits high rates of improvement as the USA and Soviet Union engaged in spirited competition. When several nations boycotted the Olympics in 1976, 1980 and 1984, the rate of improvement was reduced.

When comparing the winning women and men swimmers, the percent difference drops with distance for each stroke except for the breaststroke. In running and speed skating the percent difference rises for shorter distances and then drops off at longer distances. The swimming data is probably dominated by the greater buoyancy of women while breaststroke, running and speed skating results may depend on differences in muscle types.

Over each Olympiad, the typical rate of improvement is around two percent. Over longer periods, the improvement is much more dramatic. On July 9, 1922 Johnny Weissmuller (USA) became the first person to break one minute for 100 m in swimming. His performance as an Olympic athlete caused him to be cast in the movie role of Tarzan. About 60 years later (July 22, 1980) Vladimir Salnikov (Soviet Union) became the first to break 15 minutes for 1500 metres. Imagine a time traveler returning to the world of 1922 with the news that one man in the future could defeat a relay of fifteen Tarzans! Perhaps the most fitting conclusion regarding athletic performance lies with the motto of the Olympic movement: "Citius, Altius, Fortius" (ever faster, ever higher, ever stronger).

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THE MARRIAGE OF MATHEMATICS AND COMPUTER TECHNOLOGIES FOR SPORT IMPROVEMENT

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Abstract

The computer has had a major impact on many aspects of modern life but so far has had a limited role in sport. However the introduction of new technologies and their coming together with old technologies will introduce a new era of computerisation. The technologies that give this promise are the microcomputer, videotape and videodisc.

The data processing requirements of sport can be classified into two broad areas, the administration of a sport and the study and management of athletes. The administration area is most susceptible to the introduction of microcomputers and while many sporting bodies do use computers for low level activities they do not exploit their full potential. In the study and management of athletes technology is used well in laboratory research but not in other areas.

Likewise mathematics has great relevance to the technology behind sports equipment but does not appear to have made in-roads as an important facet elsewhere.

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1. Administrative Use

The simplest and most common administrative use of computers is for word processing and basic accounting functions. This use should increase the productivity of the office staff and give an organization better financial management. However the strain of having to keep better financial records does lead to some discontentment with having to use a computerised accounting system.

Another important administrative function that can be usefully computerised is the keeping of historical records. Introducing computers for this function requires efficient database software and a lot of energy to get the system operational. An example from an Australian Rules Football league can illustrate this point. The Frankston and District Junior Football League have about 95 games each Sunday with about 2000 players over the age groups Under 16 to Under 10, some of which have multiple grades. The League regulations require that each player have a record of the number of games played throughout the season in each grade. Hence 2000 player records need to be updated each week. This task took the secretary up to 16 hours per week. A computer system, created by the author, was introduced to facilitate this task. The system took 12 months operational testing to get it running smoothly, however the benefits have been substantial. On Sunday each team manager inputs into the computer a card for each player and then his record is automatically updated with a variety of data such as game played, goals kicked, reported by umpire, etc. The whole process typically takes only a few minutes per team. By 5:00 p.m. of each Sunday the records of 2000 players have been fully updated and the Secretary is free to spend the remainder of the week on other matters. This type of productivity tool is not welcome in all places as often volunteer staff enjoy performing record keeping tasks and have no wish to see them automated.

A complementary operation to historical record keeping is maintenance of the progressive competition records particularly scores of games and computation of ladders. Computers are valuable with this sort of work because of their computational accuracy. The Frankston League mentioned above has to compile the results of about 95 games into about 8 competition ladders. It uses the computer to such great effect that it publishes by 6:00 p.m. on the day of the competition a flawless list of scores of all games and all competition ladders.

An area where computers and mathematics can be a great aid is in the planning and design of fixtures. Typical problems in under age competitions where clubs have multiple teams are :

(i) it is better to either have all teams playing away or all at home on the one day;

(ii) play consecutive aged teams at the same venue so the older team can borrow players from the younger team if necessary;

(iii) not to draw teams that play at the same time to play on the same ground.

The author wrote a program to analyse the standard fixture tables and this knowledge now enables one to allocate teams into the tables so as to accommodate a very large number of prioritised criteria as per the examples above. It is clear that there is a great need for the application of Operational Research Methods or expert systems to assist in fixture planning.

The use of computers in sport is important as management becomes more complex. This complexity will have two deleterious effects, that is, an increasing cost of professional managers and movement of volunteers out of the activities. Computers if managed properly should give greater productivity to the manager and give more free time to the volunteer so that they can participate or spectate more, whichever is their preference.

2. Study and Management of Athletes

The study of athletes encompasses research into athletic performance in two situations, namely during a game or performance and in the laboratory in controlled experiments. Management of an athlete covers those factors effecting performance of which there are three basic categories: coaching tuition, training instruction and medical care particularly injury rehabilitation. Modern computer technologies have supported major improvements in laboratory research and medical management. However we have only just arrived at the point where in-game performance and tuition and training instruction can make big gains from the technology.

3. Study of Athletes

(i) Laboratory research

This aspect of studying athletic performance is by far the most advanced form. A wide range of biological tests are available to determine the potential of an athlete for a particular sport especially for sports that require high physical exertion. Universities, particularly throughout North America, have Departments of Kinesiology staffed with up to a dozen scientists and just as many large laboratories. The sports' scientists at the Australian Institute of Sport predominantly work in laboratory based studies. The Journal of Medicine and Science in Sport, Australia's leading sports science journal, is dominated by articles in laboratory based studies.

Computer technology is important to laboratory research in implicit and in explicit ways. Much of the biological measuring equipment, such as for measuring lung capacity or chemical analysis or body work rates, have computer circuits though the equipment as a whole would not be called a computer. The computer also serves the laboratory scientist explicitly to make statistical analyses. These analyses can be done on either a microcomputer resident in the laboratory or on a large mainframe computer. The computer has been used in the laboratory both implicitly and explicitly for well on 20 years and it will grow in its importance in this area.

Mathematicians do not appear to have gone into the sports laboratory to search out problems requiring solutions. More often the sports scientist calls on their own mathematics education or a computer package to "solve" their numerical problem.

Recently a complex mathematical model of muscle energy usage has appeared in the literature. This is clearly a sophisticated use of mathematics comparable to the design detail of yacht building. No doubt we will see more of this approach but it has its own dangers which I will comment on later.

(ii) In-Game Research

In-game research refers to research that is conducted whilst the game or athlete's performance is in progress. It can take two forms of study, firstly, taking samples from the athlete for later chemical analyses. Secondly, observing the athlete and recording the characteristics of his behaviour. This latter form of study has received a great boost from the introduction of cheap portable videotape systems and promises to make even greater advances with computer control of video cassette recorders.

The effect of coupling these technologies is a massive reduction in the work hours required to perform research of in-game performances. At Deakin University we have developed a computer controlled video recorder system. Recently the system was used to study work rates of Australian Rules footballers. A computer program was written to record the time duration of a footballer in each possible work activity such as walk, run, jog, tackle, etc. The operator watches the videotape and presses a key to represent each activity and signal its commencement. The computer monitors the time duration of each activity then produces comprehensive statistics on the player, quarter by quarter. It takes an operator about 3 hours to process a two hour game. The computer programs took a few man days work to produce. These times are to be compared with a similar hand recorded study of footballers conducted a few years ago that had the order of many man days effort to process the data from one game.

We have recently completed a project for the use of computer controlled video recorder is the analysis of judging consistency in surfboard riding competitions. This study required the use of substantial computer power for statistical analysis of scores from about 400 rides. Another proposal has recently been made to study the work rates of players in field hockey. These are examples of technology providing a means of capturing vastly greater amounts of data than ever before. It appears that mathematicians have not fully appreciated the shift in analysis emphasis this has created. I feel we are waiting for a new approach in mathematics that enables us to analyse sequences of micro-behaviour running over long periods of time.

(iii) On-Field Performance

Another type of in-game study is the analysis of the frequency and quality of the actions performed by athletes. It is traditional in team ball games to count the number of possessions and disposals of each player during the game. Recent advances in computer software have led to substantial productivity gains in this area. At Deakin University we have invented the CABER system which is used to create software for recording player behaviour in Australian Rules Football. The software represents probably a 100-fold increase in productivity in recording a game of football. Two people are needed to use the software. One person watches the game and 'narrates' it to an operator who uses a keyboard to record the narration. The keyboard has one key for each football word or phrase. The data recorded is not just the frequency of each ball handling activity, but the actual sequence of actions that make up the game. The system also permits an enormously wide range of questions to be asked of the data. In comparison a VFL club would use between 6 and 20 staff to record about one tenth of the data captured by this software.

An added option of a CABER system is the ability to pose a question to the data base, such as "Display all kicks by Peter Moore followed by mark by opposition" and then replay each event on the game videotape that satisfies the question.

The CABER system also offers significant support for other sports because it is designed as a system generator. That is, it accepts a specification of a particular game and then generates a software system tailored for that game. The system generator is known by the name CABER, Capture and Analysis of Behavioural Events in Real-time, because it can be applied to a wider range of behavioural studies than just sport. Systems have been built for waterpolo, rugby and surfing.

Other simple but quite effective computer systems have been built to assess on-field performance. The Centre for Sports Analysis at the University of British Columbia have put computers into use in the Canadian water polo and field hockey Olympic teams and have made systems for Graeco-Roman wrestling and fencing. In Britain systems have been built for squash and soccer which concentrate on recording the spatial juxtaposition of the players throughout the game.

These computing systems pose questions for mathematicians. The first question is an old one; "what data is adequately representative of an athlete's performance?" Mathematicians can contribute to this topic by providing numerical methods to assess the correspondence between subjective assessment and collected data. The second question is; "what techniques should be used to model sequential and concurrent behaviours of athletes, and how would the merits of any model be assessed?" For example, we have used Probabilistic Finite State Automata (PFSA) to model sequential behaviour of possessions and disposals in Australian Rules Football. To assess the merit of the PFSA model we analysed 1 quarter of football. In this quarter one team outscored the other by 42 points and we found statistical significance between the PFSAs of the two teams. Nevertheless, we were unable to provide any insight into what behaviours were of consequence.

Mathematics can serve sports performance well by developing analytical methods for modelling sequential and concurrent behaviours but the assessment of such research should be provided by its usefulness in the "field" rather than its mathematical "sophistication".

4. Management of Athletes

(i) Coaching Tuition

Coaching tuition appears to be the least responsive area to the introduction of new technology. This fact is no doubt due to the personal role of a coach in instructing an athlete. However, there is great potential for preparing cheaply tuitional examples from real competitions as distinct from staged examples. Indeed, television stations perform this function for advertising purposes yet no-one appears to have made a systematic collection of 'good play' for any sport. Furthermore the instructional films that are made do not have any interactive characteristic to them. Usually the athlete watches the film and is supposed to learn from the film by some sort of visual osmosis.

One of the few examples of systematic usage of videotape used to be found at Melbourne Football Club. This club had a microcomputer that controls a domestic VCR. On Sundays an operator watched a tape of the previous day's game and recorded into the computer database the appearance of a player in the picture, at the same time the computer continually monitored the tape position. The computer stored the player number and the position of the tape. It took the operator about 3 hours to play the full game tape through once and record all players' appearances. Once the recording was completed the computer was asked to replay each player's activities in turn and this replay was copied onto that player's tape. The player received the tape at training on Monday and it was used for coaching tuition of the player.

The games were accumulated on the one tape for that player so that by the end of the season the tape contained all his play for the year. This practice has discontinued with a change of coaching staff.

Richmond Football Club have a similar computer and VCR system but they concentrated on using it for planning strategy against opponents.

(ii) Training Instruction

Training instruction is restricted to those aspects of athletic training that are not the actual skills of the sport that a coach would give tuition in. Typically training instruction includes work programs designed to improve stamina or strength such as weight training, exercises and running schedules. This topic is separated from coaching tuition because one should be able to produce film and written material which covers say weight training, for a group of sports, hence the instruction is more general and not so dependent on the 'art' of the sport.

With the exception of the next topic to be discussed: medical care, this is the only area of sport where the new videodisc technology has been put to use. Mr. P. Edwards of the Liverpool Polytechnic has prepared for the British National Coaching Foundation a videodisc on weight training and another disc for the Associated Exam Board on means of assessing staff who are moderating practical sport performance. Although these projects are a small start, use of this media should expand rapidly. Unfortunately in Australia we are at the mercy of large international manufacturers who have withheld the introduction of domestic videodisc for the time being.

The important feature of videodisc is that it can access any picture out of a 60-minute film almost instantly. Hence films can be made tuitional in that an athlete can be presented with questions, and the next part of the film shown to the athlete depends on whether he answered the question correctly or incorrectly. Thus the film has the appearance of interacting the athlete's answers, hence the name "interactive videodisc'.

Mathematics at the moment does not appear to have any place in training programs.

(iii) Medical Care

The videodisc offers great potential in providing instruction for the care and rehabilitation of injuries. At the moment there is a system that uses a mannequin and videodisc system to teach resuscitation. The mannequin is wired to respond to the pressure applied by the student and the videodisc shows that part of the lesson consistent with the student's faults in performing resuscitation, that is whether the pressure is too small or too great.

Another videodisc is designed to teach anatomy and it contains considerable anatomical detail with the ability to zoom in on any part of a picture and to show the micro anatomical details of that region. In time we can expect to have instruction discs for a wide range of preventative and rehabilitation practices of sports injuries.

The role of mathematics in the application of this technology is yet to be identified. However, there is a great deal of scope for statistical analysis of injury characteristics. A survey of injuries in VFL football was carried out by Dr Hugh Seward, of the Geelong Football Club and the author. A wide range of data was collected over 4 years and the analysis although simple-minded produced changes in treatment, preventative measures and the rules of the game. A more high powered mathematical approach may well have produced even greater understanding of the nature and conditions in which injuries occur in Australian Rules Football.

5. The Future

The 1983 America's Cup win by Australia II introduced a whole new approach to sports performances that could have a major impact on human performances in the future. The approach taken by the Australian syndicate was to determine the optimum possible performance standard of their yacht. They established these standards by theoretical analysis and verified this analysis by scale model testing. The outcome of this approach was that given the wind and sea characteristics during the race, they could tell the skipper when the yacht was performing below the optimum, then the skipper had to decide what remedial action was necessary.

The modelling of human performance in individual sports, particularly track and field athletics, is a major on-going research area around the world. It is feasible that for an individual athlete, say long distance runner or swimmer, a very precise model of their theoretical performance standard could be constructed. At Deakin University a fellow engineer has designed a pocket-sized heart rate monitor that can store up to 24 hours of records. Plans for a waterproof version are underway. It is quite feasible to build the unit with a radio transmitter to send the data to a nearby computer attended by a coach, or to give output directly to the athlete. Thus, electronic monitors could be strapped to an athlete's body and give them continuous information about the gap between their theoretical and actual performance. Such an approach could provide a real tactical advantage to the individual but might also represent a real risk to one's good health if used indiscriminately. Either way it will cause a revolution in all endurance events. It is most important that mathematicians identify clearly the "credibility" and "reliability" of models created to represent human performance. At the moment the usual representation of the quality of a theoretical model is to quote the variance and the amount of variance explained by a particular parameter of interest. This information is generally uninterpretable by the athlete and the coach and often uninformative. Better explanations and evaluations need to be devised.

I would like to acknowledge the advice of my colleagues Mr. Brian Lowdon and Mr. Michael McKenna in preparing this paper.

AN ANALYSIS OF SCORING POLICIES IN ONE DAY CRICKET

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DAVID H. NOBLE[1]

Abstract

A comparison is made of the optimal scoring rates and expected scores suggested by a heuristic and a Dynamic Programming formulation in the first innings of one day cricket. Using simulation for a comparison of the distribution of runs and wickets under alternative batting strategies is discussed. The dynamic programming formulation is used to develop a method of calculating the contribution, in runs, made by each player to the team's score in a game of one-day cricket.

1. Introduction

In one-day cricket, each team has 50 six ball overs from which to score as many runs as possible. The innings of each team is terminated either when the 50 overs is completed or when 10 wickets have been lost, with the team that scores the most runs being the winner. As the chance of dismissal increases with scoring rate, there is a constant trade-off between fast scoring rates and the risk of losing a wicket. Traditionally teams preserve wickets by scoring slowly early in the innings, allowing a rapid increase in scoring rate and wicket loss in the latter overs.

Clarke[1] developed a dynamic programming formulation for the first innings in one-day cricket which showed that under an optimal policy run rate and wicket loss should be more uniform throughout the innings. This formulation calculates the optimal run rate (the rate that will lead to the largest expected score in the remainder of the innings) and the expected score in the remainder of the innings at each stage and state of an innings. The formulation is shown below.

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$$f_n(i) = \underset{R}{\mathrm{Max}} \{ P_d . f_{n-1}(i-1) + R/6 + (1-P_d). f_{n-1}(i) \}$$

where $f_n(i)$ = expected score in remaining n balls with i wickets in hand.

i = wickets in hand, i = 0 to 10.

n = balls to go, n = 0 to 300.

P_d = probability of dismissal per ball.

R = runs per six ball over.

The relationship between the run rate and the probability of dismissal that will be used throughout this paper is given in Table 1. In addition to the probability of dismissal P_d for each run rate the average number of balls faced before dismissal $1/P_d$ and the expected partnership size $(1/P_d).(R/6)$ are given for that run rate. This value gives a better measure of the difficulty of scoring at that run rate. Using this relationship $f_n(i)$ can be calculated to give for each value of balls to go and number of wickets in hand the optimal run rate and the expected score in the remainder of the innings. The results for selected values of wickets in hand and overs to go are shown in tables 2 and 3.

Clarke suggested a simple heuristic that could be used by batsmen to approximate the optimal run rate, and also suggested using as a measure of performance the *extra* runs each player has contributed to the score. This paper further investigates these possibilities.

TABLE 1: Probabilities associated with given run rates.

Run Rate R	Probability of dismissal Pd	Expected number of balls before dismissal	Expected partnership size	Probability of scoring one run P_1	Probability of scoring two runs P_2	Probability of scoring zero runs P_0
1.0	0.005	200	33.3	0.167	0.000	0.829
1.5	0.008	128	31.9	0.250	0.000	0.742
2.0	0.011	92	30.6	0.333	0.000	0.656
2.5	0.014	70	29.2	0.417	0.000	0.569
3.0	0.018	56	27.8	0.500	0.000	0.482
3.5	0.022	45	26.4	0.583	0.000	0.395
4.0	0.027	38	25.0	0.667	0.000	0.306
4.5	0.032	31	23.6	0.750	0.000	0.218
5.0	0.038	27	22.2	0.833	0.000	0.129
5.5	0.044	23	20.8	0.917	0.000	0.039
6.0	0.051	19	19.4	0.000	0.500	0.449
6.5	0.060	17	18.1	0.000	0.542	0.398
7.0	0.070	14	16.7	0.000	0.583	0.347

TABLE 2: Optimal run rates.

Overs to go	Wickets in hand 1	2	3	4	5	6	7	8	9	10
1	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0
5	4.5	6.5	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0
10	3.0	4.5	5.5	6.5	7.0	7.0	7.0	7.0	7.0	7.0
15	2.5	3.5	4.5	5.5	6.0	7.0	7.0	7.0	7.0	7.0
20	2.0	3.0	4.0	4.5	5.0	6.0	6.5	7.0	7.0	7.0
35	1.5	2.5	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
30	1.5	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.0
35	1.5	2.0	2.5	3.5	4.0	4.0	4.5	5.0	5.5	5.5
40	1.0	2.0	2.5	3.0	3.5	4.0	4.5	4.5	5.0	5.5
45	1.0	1.5	2.5	3.0	3.0	3.5	4.0	4.5	4.5	5.0
50	1.0	1.5	2.0	2.5	3.0	3.5	3.5	4.0	4.5	4.5

TABLE 3: Expected innings scores under optimal policy

Overs to go	Wickets in hand 2	4	5	6	7	8	9	10
1	6.9	7.0	7.0	7.0	7.0	7.0	7.0	7.0
5	25.7	33.8	34.7	35.0	35.0	35.0	35.0	35.0
10	36.6	56.8	62.9	66.6	68.5	69.4	69.8	69.9
15	42.8	70.5	80.7	88.9	95.2	99.5	102.1	103.6
20	47.0	80.0	92.9	104.0	113.5	21.5	127.8	132.5
25	50.1	87.1	102.2	115.4	127.2	37.5	146.6	154.3
30	52.5	92.7	109.5	124.5	138.0	50.2	161.1	171.0
35	54.3	97.2	115.5	132.0	147.0	60.7	173.2	184.6
40	55.9	100.9	120.4	138.2	154.6	69.6	183.4	196.1
45	57.2	104.1	124.7	143.6	161.1	77.2	192.2	206.1
50	58.2	106.8	128.3	148.2	166.7	83.9	199.9	214.9

2. Heuristic versus DP rates

The suggested heuristic selects the run rate that will result in all wickets being lost as near as possible to the end of the 50th over, i.e. a wicket loss every (balls to go / wickets in hand) balls. For instance if there are 120 balls (20 overs) remaining in the innings and 6 wickets in hand then the heuristic suggests losing a wicket every 120/6=20 balls. Table 1 shows that the run rate that would result in the loss of a wicket in as close to 20 balls as possible is 6 runs per over. Therefore if there were 120 balls remaining in the innings and 6 wickets in hand the heuristic suggests a run rate of 6 runs per over, the same rate as given by table 2.

The heuristic's validity is tested by comparing the run rates suggested by the heuristic with those calculated by the dynamic programming formulation. Table 4 shows the difference between the heuristic and the dynamic programming run rates as given in table 2. The run rates selected by the heuristic and dynamic programming formulation are almost identical for the selected states and stages.

TABLE 4: Graphical comparison of DP and heuristic run rates

Overs to go	Wickets in hand									
	1	2	3	4	5	6	7	8	9	10
1										
5		+								
10			+	+						
15	-				+					
20					+					
25										
30		-								+
35	-			-	-					+
40		-								
45			-	-						
50										

: a blank indicates agreement
+ : the heuristic is 0.5 runs/over above the optimal
- : the heuristic is 0.5 runs/over below the optimal

Another test of the heuristic's ability is to compare the expected innings scores of teams scoring at heuristic run rates with the expected innings scores of teams scoring at dynamic programming run rates. In this case, the maximum difference (in runs) in the expected scores between the heuristic and dynamic programming was 0.3 runs. The closeness of the expected innings scores under the dynamic programming formulation and heuristic confirms the conclusion that the heuristic is a valid method of selecting near optimal run rates.

Although all of the comparisons between the heuristic and dynamic programming formulation shown here have been made under the one relationship between run rate and probability of dismissal (as shown in table 1), several other relationships have been tested with almost identical results. The maximum difference between the totals under the two policies was less than 4 runs.

3. Simulation of scores under several scoring policies

To obtain an insight as to why batsmen choose strategies other than the Dynamic Programming model suggests, simulation was used to determine the distribution of scores under different batting policies. The policies compared were the current scoring policy (as determined by empirical data), the dynamic programming policy and a policy of scoring 4 runs every over. Five hundred trials of each policy were run, using the same random number stream for each innings under the three policies. The relationship in Table 1 was used for all simulations.

A batsman attempting to score at 1 run per over will not score 1/6 runs off every ball but will score 0 to 6 runs off each ball in a distribution which results in an average of 1/6 runs per ball. If P_k is the probability of scoring k runs off a particular ball where $0<=k<=6$, then $\sum kP_k = R/6$. To simplify the calculations only one value of P_k (for either k=1 or 2) other than P_0 was assumed to be non zero. Once this value k* is found, P_{k*} is given by $P_{k*} = R/(6k^*)$ and the value of P_0 can be determined as $P_0 = 1 - P_d - P_{k*}$. The final three columns of table 1 show the transition probabilities that were used during the simulation

To determine the run rate policy actually used by cricketers, data was collected from about 30 innings over 3 series of international matches held in Australia between 1989 and 1992. Run rates were calculated in groups of 5 overs for each number of wickets in hand. Moving averages of 3 were used across run rates and down overs to smooth the data. The resultant run rates are shown in table 5.

The results at this stage are a little inconclusive. The negative skewness of the number of runs scored under the DP policy may be a reason why cricketers choose other policies, but further work is needed in this area.

TABLE 5: Run rates actually used.

overs to go	Wickets in hand 1	2	3	4	5	6	7	8	9	10
1-5	3.0	4.5	5.0	5.5	6.0	6.5	6.5	7.0	7.0	7.0
6-10	2.5	3.5	4.0	4.5	5.0	5.0	5.5	5.5	6.0	6.5
11-15	2.0	2.5	3.5	4.0	4.5	4.5	4.5	5.0	5.5	6.5
16-20	2.0	2.5	3.0	3.5	3.5	4.0	4.0	4.5	5.0	6.5
21-25	2.0	2.0	2.0	3.0	3.0	3.5	4.0	4.0	5.0	6.0
26-30	2.0	2.0	2.0	2.5	3.0	3.0	3.5	3.5	4.5	5.0
31-35	2.0	2.0	2.0	2.5	2.5	3.0	3.0	3.0	4.0	4.5
36-40	2.0	2.0	2.0	2.5	2.5	2.5	2.5	3.0	3.5	3.5
41-45	2.0	2.0	2.0	2.5	2.5	2.5	2.5	3.0	3.0	3.5
46-50	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.5	3.0

4. Performance measures

In this section we investigate the development of radically different performance measures. Although it is generally recognised that one-day cricket is quite different from test cricket, the measures of performance usually used are the same - batting average (runs scored / times wicket lost) and bowling average (runs scored / wickets taken). In addition a strike rate for batsmen (runs scored / balls faced) and strike rate for bowlers (balls bowled / wickets taken) are often quoted. Here we use as a measure the extra runs scored by the batsman (or off the bowler) over that expected under some criteria. Thus each ball, the batsman's net contribution to the score is the expected total team score after the ball has been bowled less the expected total score before the ball was bowled. A bowler's contribution is the negative of the above, and the contributions for each ball are accumulated through the innings. While in this paper we use the optimal scoring rate suggested by the dynamic programming formulation, the method could also be applied using (say) scoring rates supplied by coaches and team management. Johnston and Clarke [2] give a full account of the method and some results.

For example, if the expected score in the remainder of the innings with 50 balls to go and 5 wickets in hand is $f_{50}(5) = 70.0$ and the expected score in the remainder of the innings with 49 balls to go and 5 wickets in hand is $f_{49}(5) = 68.50$, then the batsman on strike when 50 balls remain in the innings must score 1.5 runs for the side to have the same expected innings score. If the batsman scores 2 runs in this situation then he has advanced his teams expected innings score by 0.5 runs. In this case the batsman's performance measure would increase by 0.5 and the bowler's measure would decrease by 0.5.

Because the measures for bowlers and batsmen are essentially the same (extra runs over what is expected) they can be added and so the performance measures for each player in a match is the sum of his batting and bowling performance measures in each innings.

The calculation of the values of $f_n(i)$ using equation 1 requires that the relationship between R and P_d must be input into the dynamic programming formulation. It was decided to develop a relationship between R and P_d which was 'fair'. In other words a relationship which when used to calculate performance measures did not favour batsman over bowlers or vice-versa. Each player could then be measured against a known standard. Several relationships were developed and used to calculate the performance measures on actual matches, using ball by ball data derived from traditional scoresheets. The relationship shown in Table 1 was considered the best in producing measures that agreed with a subjective appraisal of performance.

Results

Table 6 shows the performance measures for all of the Australian players in the match played in Melbourne on 26/12/89 between Australia and Sri Lanka. Australia won this match by scoring 5 for 228 to Sri Lanka's all out for 198.

TABLE 6 - Performance measures for match on 26/12/89.

First Innings - Australia batting

Batsman	Runs	Balls	Performance Measure
M.Taylor	11	25	-10.1
G.Marsh	38	82	2.9
D.Boon	11	19	-7.8
D.Jones*	85	89	34.9
A.Border	11	15	-5.6
S.Waugh	5	7	-7.8
S.O'Donnell*	57	60	10.7
Extras	4		4.0

TABLE 6 (cont) - Performance measures for match on 26/12/89.

Second innings - Sri Lanka batting

Bowler	Overs	Wkts	Runs	Performance Measure
M.Hughes	9.2	2	41	6.1
G.Campbell	10	0	36	-8.1
S.O'Donnell	9	4	36	19.1
S.Waugh	6	1	26	1.1
P.Taylor	10	2	36	7.8
A.Border	3	0	17	-4.3
Run Outs	-	1	-	8.5

* = not out

Batting & bowling combined

Player	Batting Performance Measure	Bowling Performance Measure	Total Performance Measure
D.Jones	34.9	-----	34.9
S.O'Donnell	10.7	19.1	29.8
P.Taylor	-----	7.8	7.8
M.Hughes	-----	6.1	6.1
G.Marsh	2.9	-----	2.9
S.Waugh	-7.8	1.1	-6.7
D.Boon	-7.8	-----	-7.8
G.Campbell	-----	-8.1	-8.1
A.Border	-5.6	-4.3	-9.9
M.Taylor	-10.1	-----	-10.1

The performance measures suggest that there were two players whose performance was well above the others, Dean Jones (performance measure of 34.90) and Simon O'Donnell (29.78) The 'man of the match' award which is presented to the player that the commentators feel was the best player for that match was given to Simon O'Donnell who was selected as the second best player by the performance measures. Of course the adjudicators may have included an allowance for fielding that the performance measure ignores. It is of interest that it was in fact O'Donnell who ran out the Sri Lankan batsman, which table 2 shows cost Sri Lanka another 8.5 runs.

In international matches the performance of players is closely monitored by the public and selectors. The interpretation of the usual measures would always be tempered by intimate knowledge of how the runs were scored, at what stage wickets fell, etc. This is not the case with many lower level matches. With computer based ball by ball scoring, methods such as the above could easily be incorporated to produce player performance measures that truly reflected a player's contribution.

References

[1] S. R. Clarke, "Dynamic Programming in One-Day Cricket - Optimal Scoring Rates", *Journal of the Operational Research Society* **39**, No. 4 (1988) 331-337.

[2] M. I. Johnston and S. R. Clarke, "Assessing player performance in one day cricket using Dynamic Programming". To appear in *Asia-Pacific Journal of Operational Research.*

COMPUTER AND HUMAN TIPPING OF AFL FOOTBALL - A COMPARISON OF 1991 RESULTS

STEPHEN R CLARKE[1]

Abstract

For over a decade the author has been involved in computer tipping of VFL and now AFL football. Evidence suggests that the computer, although ignoring much information available to human tipsters, is at least as accurate. This paper explores the difficulty of predicting, analyses the accuracy of the computer in 1991, compares the relative accuracy of human and computer tipping in 1991, and investigates some reasons for limiting human performance.

1. Introduction

In 1981 The Sun News Pictorial began publishing the results of a computer tipping program written by the author. This continued until 1986, when The Sun decided to concentrate on human tipsters. Some details of this period are contained in Clarke [1],[2]. In 1991 The Age published the now updated computer program tips for winners and margins along with the predictions of winners by several experts. The Sun meanwhile published both the predicted winners and margins for 12 experts and 12 celebrities. This allows an opportunity to compare the accuracy of the computer with those of so called experts, and the general public.

Details of computer methods for tipping football are contained in Clarke [3], Harville [4], Stefani [6],[7],[8] and Stefani & Clarke [9]. The program discussed here uses an exponential smoothing algorithm, to produce team ratings and team/ground interaction factors for each team. Of relevance to the present paper is that the algorithm uses only the names of the teams playing, the ground the match is played on, and the previous final results of the matches. It ignores all other data, many of which the average and expert follower believe is important. The computer knows nothing of such things as team personnel (absence of key players), weather, time of day (eg night matches), previous team played (eg bye), time since last match, etc. One would therefore expect the humans to out-perform the computer.

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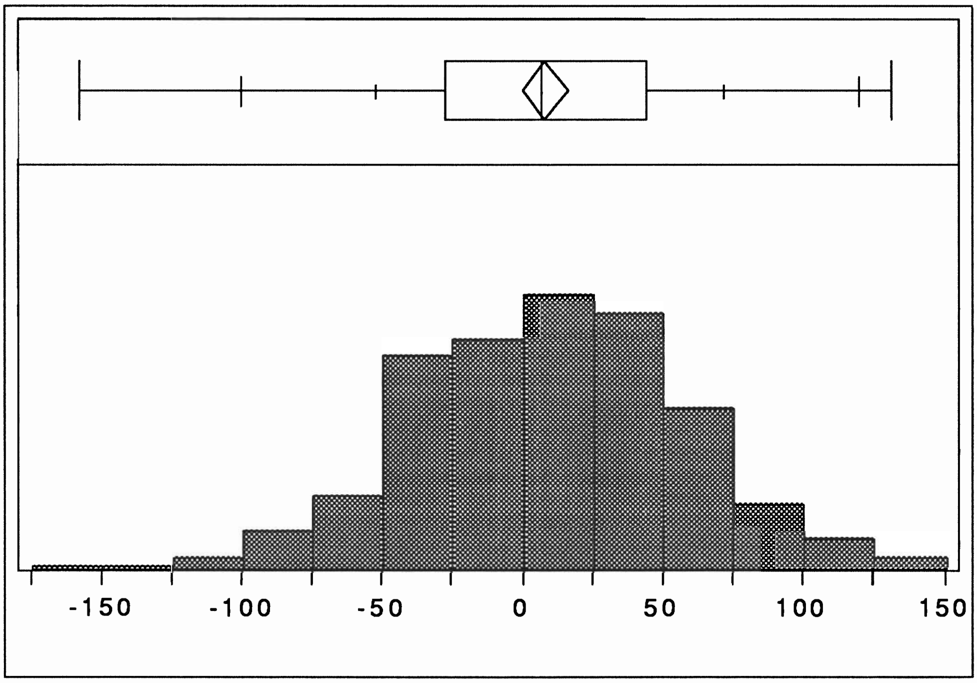
2. Distribution of Margins

Before looking at how the computer has performed, it is worth looking at how difficult the task has become. Figure 1 shows the home ground margins for home and away matches. The distribution of scores is reasonably symmetric. The mean home ground advantage for the (nominal) home teams is 8.3 points. Note the large spread of scores - standard deviation of over 50 points. Stefani and Clarke [10] show that prediction of winners in football has become more difficult in the latter half of the eighties. In terms of margins this is even more apparent. A comparison of 1980 and 1981 absolute margins is shown in Figure 2. Clearly the proportion of large winning margins has increased. Most percentiles have increased by 10-20%, with both the mean and median margins increasing by over 7 points.

FIGURE 1. Distribution of home team winning margins - 1991



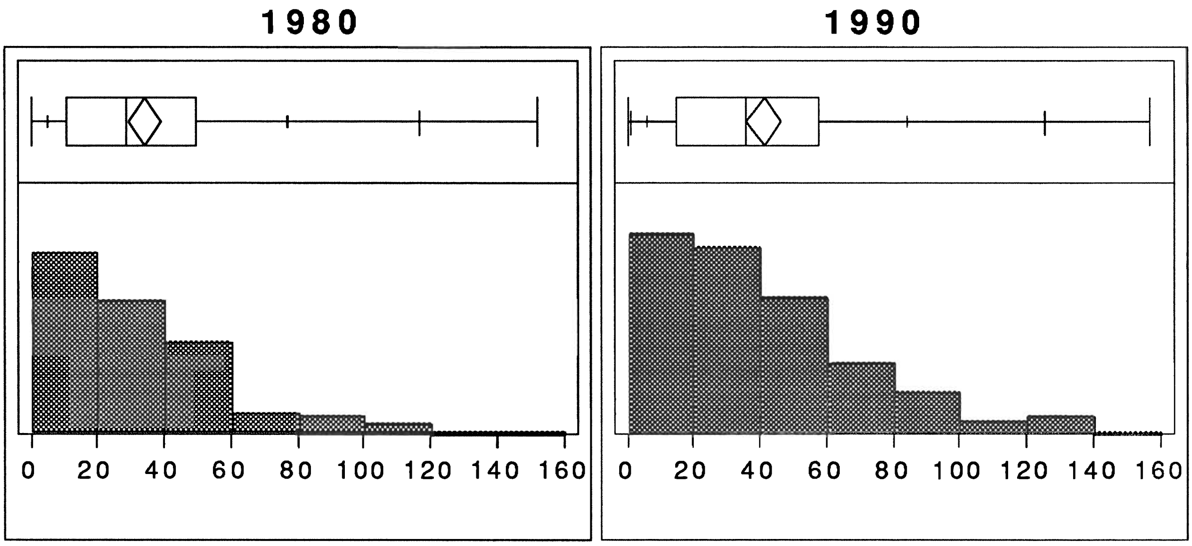
Quantiles

maximum	100.0%	131.00
	90.0%	72.40
quartile	75.0%	44.00
median	50.0%	7.00
quartile	25.0%	-27.50
	10.0%	-52.00
minimum	0.0%	-157.00

Moments

Mean	8.3697
Std Dev	51.6123

FIGURE 2. Comparison of absolute margins in 1980 and 1990



Quantiles

	1980			1990	
maximum	100.0%	152.00	maximum	100.0%	157.00
	97.5%	116.70		97.5%	125.55
	90.0%	77.00		90.0%	84.80
quartile	75.0%	49.75	quartile	75.0%	58.00
median	50.0%	29.00	median	50.0%	36.00
quartile	25.0%	11.00	quartile	25.0%	15.00
	10.0%	5.00		10.0%	6.00
minimum	0.0%	0.00	minimum	0.0%	0.00

Moments

	1980		1990
Mean	34.2500	Mean	41.2667
Std Dev	29.4474	Std Dev	32.0021
N	132.0000	N	165.0000

Selecting matches with the greatest margins gives a possible reason for the change. The matches with the greatest winning margins (over 75 points) are shown in Table 1. Eighteen out of 21 of these matches involve an interstate team - an affect entirely absent when the author started tipping. (In addition, the round 21 match was actually played in Tasmania).

TABLE 1. Matches resulting in a margin greater than 75 points

Round	home	away	RESULT
1	ADEL	HAWT	86
1	WCE.	MELB	79
2	HAWT	SYD.	91
4	BRIS	GEEL	-102
7	HAWT	WCE.	-82
7	ST.K	ADEL	131
8	FITZ	SYD.	-77
9	WCE.	FITZ	99
11	GEEL	ADEL	84
13	WCE.	FOOT	118
13	HAWT	BRIS	87
14	COLL	SYD.	99
15	COLL	ADEL	123
15	SYD.	MELB	-83
17	WCE.	COLL	81
19	GEEL	BRIS	101
20	BRIS	COLL	-101
21	HAWT	FITZ	126
23	CARL	HAWT	-96
23	ST.K	BRIS	120
24	ESS.	HAWT	-80

3. Prediction Accuracy

Winners

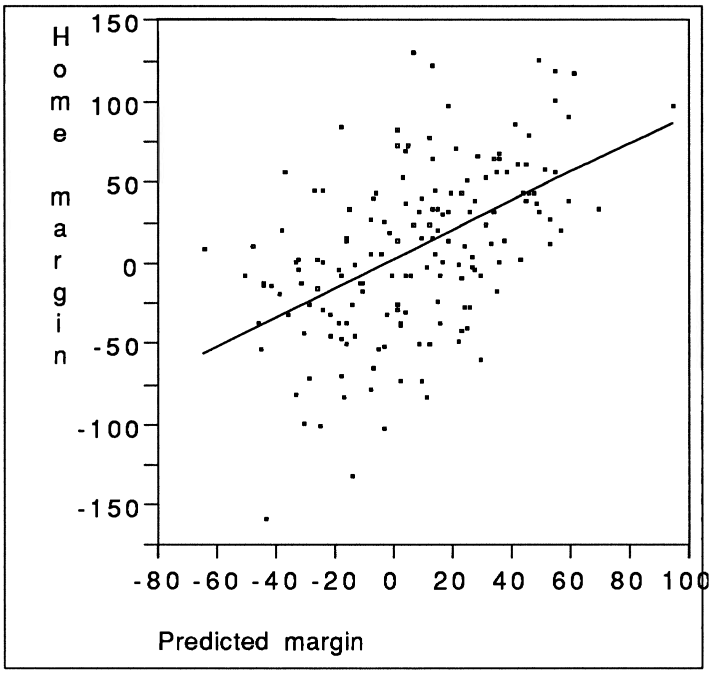
In 1991 the computer correctly selected 116 winners out of 165 home and away matches, and 5 out of 7 finals. At just over 70% correct this is slightly better than the decade average for a computer tip reported in Stefani & Clarke [10].

Margins

Figure 3 shows the relationship between the predicted and the actual margin. The fit accounts for about 25% of the variation. Given that the prediction takes account of team ability, current form and ground advantage there is still a large degree of unexplained or random variation. Computer predictions, because they are predicting the expected score, will never have the variation shown by the actual values. Figure 4 demonstrates this, but also gives an idea of the spread of results for predictions in given ranges.

We now look at the distribution of errors, defined as the difference between forecast and actual home ground margin. Figure 5 shows the distribution of errors. Note that mean error is still slightly negative although not significantly so, and the median error is -5.00. This implies that the home advantage is possibly not large enough - the computer may still be adjusting to interstate teams and their large home advantage. The table shows the median absolute error is 30, with a mean of 36. Thus half the time the computer is less than 5 goals out (or pessimistically, half the time it is more than 5 goals out).

FIGURE 3. Actual margin versus predicted margin



Summary of Linear Fit

Rsquare	.2589047
Root Mean Square Error	44.59554
Mean of Response	8.357575

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	2.1448566	3.56804	0.60	0.5486
Predicted margin	.89372157	.118433	7.55	0.0000

FIGURE 4. Distribution of actual margins for ranges of predicted margins.

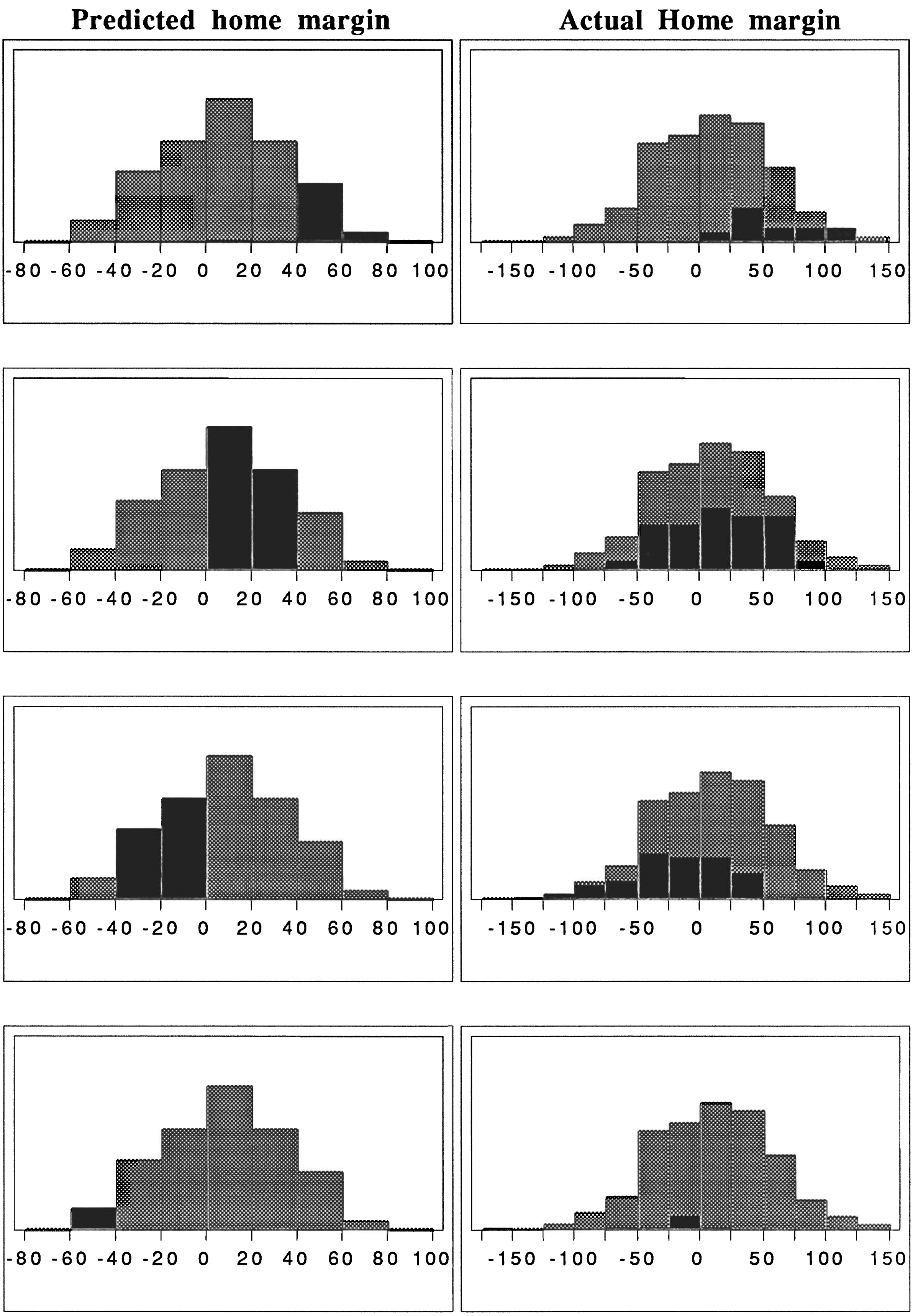
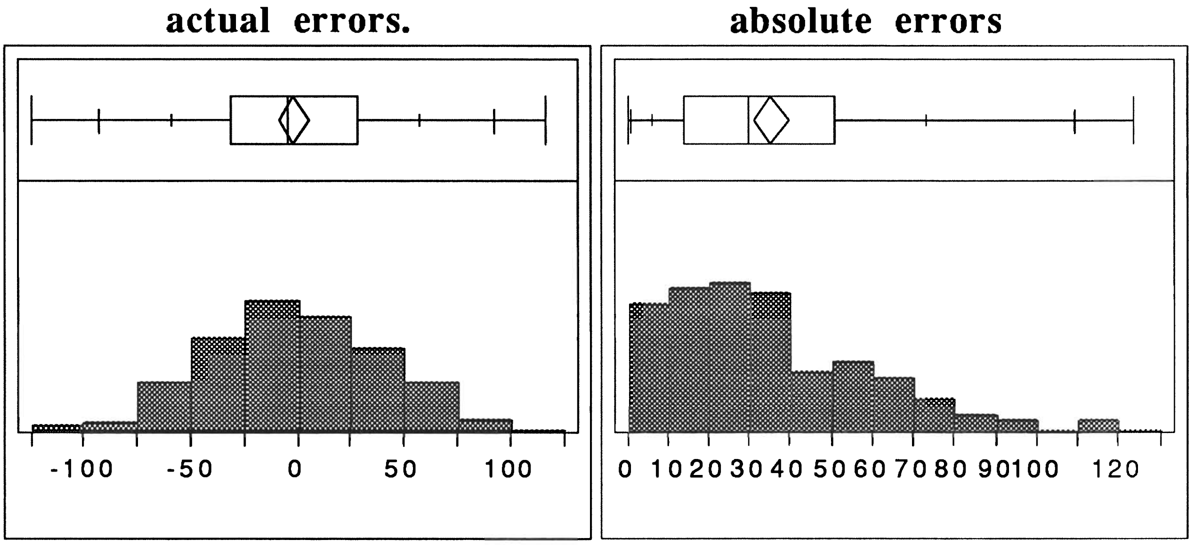


FIGURE 5. Distribution of errors.



Quantiles

maximum	100.0%	117.00	maximum	100.0%	124.00
	90.0%	57.40		90.0%	73.40
quartile	75.0%	29.00	quartile	75.0%	51.00
median	50.0%	-5.00	median	50.0%	30.00
quartile	25.0%	-30.50	quartile	25.0%	14.00
	10.0%	-59.00		10.0%	6.00
minimum	0.0%	-124.00	minimum	0.0%	0.00

Moments

Mean	-1.4061	Mean	35.4424
Std Dev	44.5691	Std Dev	26.9177
N	165.0000	N	165.0000

Final ladder predictions

Although not usually published, the computer also predicts each round the final ladder at the end of the home and away season. Given the intricacies of the draw, this is one area where the computer should have advantages over human tipsters. Unfortunately, expert predictions of final ladder position are usually only published at the beginning of the season. Table 2 shows the final ladder predictions before each of the 24 rounds. The teams are in order of actual finishing position. The computer clearly has more trouble with the middle of the ladder rather than the very top and bottom. Defining a prediction to be close if within one of the true final position, the final row shows the steady improvement through the season. After 4 rounds over half the teams are placed closely, and by round 17 about 12 out of 15 teams are closely placed.

Because ladder position can alter drastically due to just one game, it is also worth looking at predicted final premiership points. Again, if we look at a close prediction as within 4 premiership points (1 game), the final row shows that from round 16 onwards the computer has closely placed almost all the teams.

TABLE 2. Predicted Final Position by Round

Symbol is the first letter of the team name

```
         |
         |
     15 +    s s s s f f f f f f f f f f f f f f f f f f f f B
P        |
o    14 +    R R f f B B B B B B B B B B B B B B B B B B B B f
s        |
i    13 +    f f R B R R s R R R R s s R s R R s s R R R R R R
t        |
i    12 +    B B B R s s R s s s s R R s R s s R R s s s s s s
o        |
n    11 +    G G F F F F F F F F F C C F F A F F c c F c A c c
         |
     10 +    F F G G S N N A A A C F F C C F A c F F A A c F F
         |
      9 +    S M S S N A S G G C G A A c A M c A A A c F F A A
         |
      8 +    A S A M A S A N c c A c c A M C M M C C C M C N N
         |
      7 +    W c E A c c c c N G c G N M c c C C M M N C N C C
         |
      6 +    M E M N G G G S C N N N G N N N N E N E E E M M E
         |
      5 +    c W c c M M C H S S S M M G G E E N E S M S E E M
         |
      4 +    E A N E C H M C M E M S H H H G G S S N S N S S S
         |
      3 +    N N W W E C E M H M E H S E E H H H H H H G G G G
         |
      2 +    C H C C W E H E E H H E E S S S S G G G G H H H H
         |
      1 +    H C H H H W W W W W W W W W W W W W W W W W W W W
         |
        ---+---------+---------+---------+---------+---------+--
ROUND      0         5         10        15        20        25

number >1        1   1
position away  9 1 9 1 6 7 6 7 3 6 5 5 5 8 8 6 4 2 3 3 2 2 2 0 0

number >1      1 1   1 1   1
game away      0 1 9 1 0 9 0 9 5 6 3 6 5 5 7 3 4 4 1 2 1 1 0 0 0
```

4. Comparison with human tipsters

Table 3 shows the number of correct winners and percentage correct for all the tipsters in The Age and The Sun. In some cases (such as leader of the Opposition) selections from different people have been combined. Draws are counted as half correct. For the home and away matches the computer correctly selected 116 winners out of 165 matches, a success rate of 70.3%. Of The Age tipsters, the nearest to this was Ron Carter with 111 or 67.3%. Only 2 of The Sun experts, and one of the celebrities beat the computer, with another 2 celebrities choosing the same number of winners. In interpreting a table such as this, it should be borne in mind that in selecting 165 matches, each with a probability of success of .7, the number of correct choices will have a standard deviation of about 6. As the computer gives its own estimate p_i of the probability of success for each prediction, the mean and variance of the number correct over the season is $\sum p_i = 121.7$ and $\sum p_i(1-p_i). = 29.35$, giving a standard deviation of 5.4. Thus by the computer's own estimates it had an unlucky year. (In fact the high value of $\sum p_i$ is probably an indication that the probability estimates need updating. With the general increase in margins as discussed earlier, a predicted win of 20 points (say) implies a lesser chance of winning than it did 10 years ago. Thus the computer is probably over estimating the chance of selected teams winning). I suspect that differences between commentators in number of winners less than about 5 are probably insignificant. Nevertheless, the general public don't see it this way, and it is better to be top of the table than bottom.

Table 3 also shows the total and average absolute errors of the margin predictions for The Sun tipsters. Only one expert and one celebrity performed better than the computer. (Although perhaps the computer is more intelligent than we give it credit for, and thought it politic to come in just behind the Prime Minister).

Reasons for computer supremacy

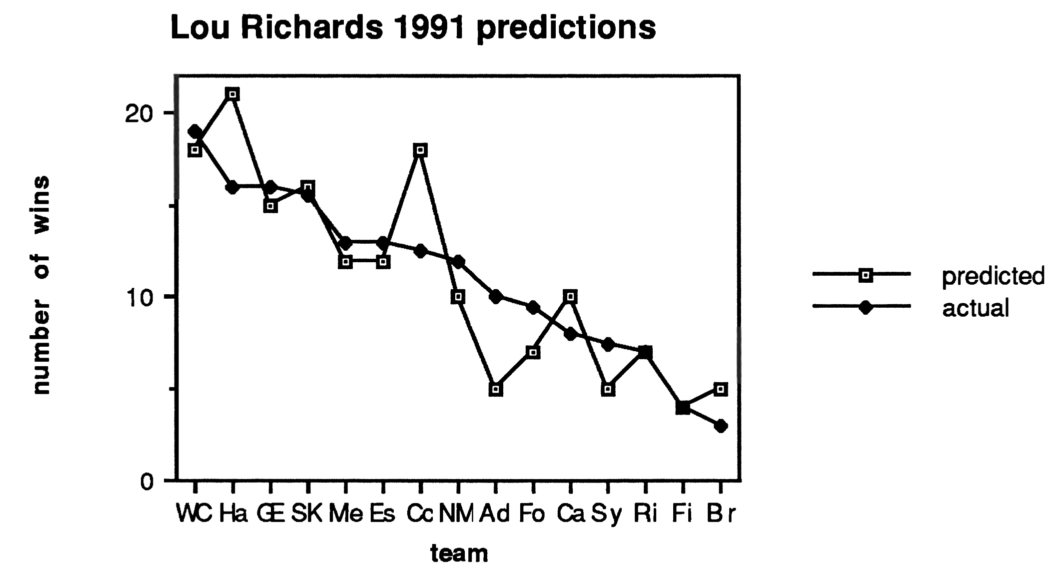
Figure 4 shows that the distribution of the computer margin prediction is roughly the same shape as that of the actual margins, with the same mean but a lesser variance. This is not true of many human tipsters, who often have a distinctly bimodal distribution of predicted margins. There appears to be an aversion to predicting close margins. In addition, some tipsters tend to choose mutiples of 10 or 6 points for the margins. One reason the computer may perform better than experts is that it has no loyalties to particular teams. While no data is available on the teams followed by many of the experts, there is evidence to suggest that tipsters are certainly influenced (to their detriment) by the teams they follow. Figure 6 shows a graph of the number of times Lou Richard's selected each team and the number of wins for each team. Clearly Lou favours Collingwood, the team he barracks for. This graph is typical of all the celebrities.

TABLE 3. Accuracy of The Age and The Sun tipsters.

Tipster	Number tipped	Number correct	Percentage correct	Total Deviation	Average Deviation
Computer	165	116	70.3	5848	35.4
Age experts					
Ron Carter	165	111	67.3		
Greg Baum	110	74	67.3		
Nick Johnson	76	51	67.1		
Gary Linnel	74	49.5	66.9		
Martin Blake	153	102	66.7		
Steve Linnel	102	67.5	66.2		
Len Johnson	156	103	66.0		
Penny Crisp	95	62.5	65.8		
Patrick Smithers	55	36	65.5		
Peter Schwab	7	3.5	50.0		
Sun Experts					
Geoff Poulter	158	115	72.8 *	5476	34.7 *
Ron Reed	158	109	69.0	5702	36.1
Ron Barassi	165	117	70.9 *	5898	35.8
Bruce Matthews	158	109	69.0	5611	35.5
Niall/Pierce	165	113	68.5	6333	38.4
Don Scott	165	111	67.3	6040	36.6
Tony De Bolfo	165	110	66.7	6038	36.6
Daryl Timms	165	109	66.1	6135	37.2
Crackers Keenan	165	107	64.9	5941	36.0
Michael Stevens	165	107	64.9	6170	37.4
Lou Richards	165	103	62.4	6514	39.5
Eva/Atkins/West.	158	101	61.2	5750	36.4
Sun Celebrities					
Joan Kirner	165	118	71.5 *	5909	35.8
Bob Hawke	165	116	70.3	5839	35.4 *
Wynne/Meldrum	165	116	70.3	5943	36.0
David Johnston	165	113	68.5	6111	37.0
John Hewson	165	112	67.9	6019	36.5
Daryl Somers	165	111	67.3	6001	36.4
Mary Delahunty	165	110	66.7	6223	37.3
Steve Vizard	165	104	63.0	6455	39.1
Brown/Kennett	165	98	59.4	6863	41.6

* Better performance than the computer

FIGURE 6 Lou Richards' predicted and actual number of wins for each team.



With the exception of Bob Hawke, all celebrities selected the team they followed more often than they won, the excess ranging from 5 to 9 wins.

It is well known that supporters look for any reason to convince themselves that their team will win next week. Nevertheless it is interesting that football followers predict most poorly the performance of the team they know most about. One reason humans may choose poorly is that they know too much information, and they overrate the importance of much of it. The return of a player from absence due to injury, good training form, a perceived after effect of a bye, etc might also be given too much weight by experts. However all the experts share much the same information. Morrison & Schmittlein[5], show that 10 experts whose forecasts show a correlation of 0.6 are equivalent to only 1.56 independent forecasts. It would be interesting to look at the correlations between the margin tips of experts, to see if the tips of those with shared information (such as expert tipsters from The Sun), are more closely correlated within groups than between groups.

5. Conclusion

An analysis has shown the computer's performance in predicting the winner and margins in 1991 was better than the average expert or football follower. The computer uses only the previous match results and is not influenced by publicity surrounding particular events, nor club loyalties. As such it is likely to be more independent than the experts, and the single computer tip may provide more extra information to followers than the many additional human experts.

Computer forecasts of sporting events provide an interesting, objective and useful alternative to the human expert.

Acknowledgements

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A MATHEMATICAL MODEL OF GIANT SWINGS ON THE HORIZONTAL BAR

FRANK GATTO[1] and ROBERT NEAL[2]

Abstract

A mathematical model of a gymnast performing giant swings on the high bar is presented. The gymnast is modelled as a single rigid link rotating about the high bar. As well, the lateral stiffness of the bar is considered and modelled as two linear springs. Friction between the hands and the bar is also considered. The motion of this system is described by a set of three nonlinear equations which are solved numerically by assuming constant average acceleration between each time step. Several examples are presented which examine the mechanical behaviour of the system.

1. Introduction

This paper is part of a study which is investigating the factors which cause injuries to the wrists of gymnasts performing backward giant swings on the high bar. Wrist injuries are a common complaint for gymnasts at all levels of competitive involvement. A recent report of 38 collegiate gymnasts in the U.S.A. found that 75 percent of males and 33 percent of females in the study had experienced wrist pain over a four month period (Mandelbaum et al. [5]). This is understandable since a large proportion of the skills needed for gymnastic training require weight bearing through the arms and wrists and in most cases, wrist pain is related to the compressive forces acting through the wrist as a result of weight bearing on the hyper-extended wrist (Read [7], Roy et al. [8]).

Kopp and Reid [4] measured via a series of experiments the forces and torques generated at the gymnast's hands while they performed backward and forward giant swings. The high bar was instrumented with strain gauges to enable the measurement of horizontal and vertical forces and torques. The manoeuvres were filmed at 80 frames per second. The force time and torque time history at the hands of the gymnast were obtained. They found that for backward giant swings, the maximum forces generated ranged from 3.45 to 3.7 times the body weight with a mean value of 3.57 times the body weight. The maximum force consistently appeared in the third quadrant of the swing just past the bottom of the swing.

A biomechanical model of the giant swing can be used to predict the forces across

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the wrist and to help obtain a better understanding of the mechanical principles involved in the proper execution of the giant swing. A mathematical model of a gymnast performing giant swings on the high bar is presented. This model predicts the movement of the gymnast and the forces and torques generated at the gymnast's hands.

2. Mathematical Model

The gymnast is modelled as a single rigid link of mass, m_g, and inertia, I_g (given about the C.G. of the gymnast), rotating about one end, i.e. the gymnast hands attached to the high bar. The length from the gymnast's hands to the centre of gravity is given by r_g. The dynamic friction between the gymnast's hands and the bar is modelled as a torque and is given as a constant, k_f, times the force normal to the bar at the hands of the gymnast. This torque acts in the direction opposing motion. The high bar is modelled as a single mass, m_b, attached to rigid supports by horizontal and vertical linear springs, k_{bH} and k_{bV} respectively. These springs represent the lateral bending stiffness of the high bar and are given by (Todd [9])

$$k_{bH} = k_{bV} = k_b = \frac{48 E I_b}{l_b^3} \tag{1}$$

where

- l_b = length of the high bar;
- E = modulus of elasticity of the high bar;
- I_b = area moment of inertia;
  = $\pi d^4 / 64$ (for a solid circular section);
- d = diameter of the high bar.

Figure 1 shows a representation of the mathematical model proposed and the forces acting on the system including the d'Alembert forces, the forces due to the lateral stiffness of the high bar and the friction forces between the gymnast's hands and the bar. The following nomenclature is used

- x_b = horizontal deflection of the bar;
- y_b = vertical deflection of the bar;
- k_b = lateral bending stiffness of the bar;
- m_b = the mass of the bar that is accelerated - half the total mass of the high bar;
- θ_g = rotation of the gymnast;
- x_g = horizontal position of the centre of mass of the gymnast;
- y_g = vertical position of the centre of mass of the gymnast;
- I_g = inertia of the gymnast about the centre of mass;
- r_g = length from the high bar to the centre of mass of the gymnast.

The motion of this system can be described totally by three generalised variables. These are the horizontal deflection of the bar, x_b, the vertical deflection of the bar, y_b, and the rotation of the gymnast, θ_g. Therefore, it is possible to formulate three independent equations which fully describe the motion of the system.

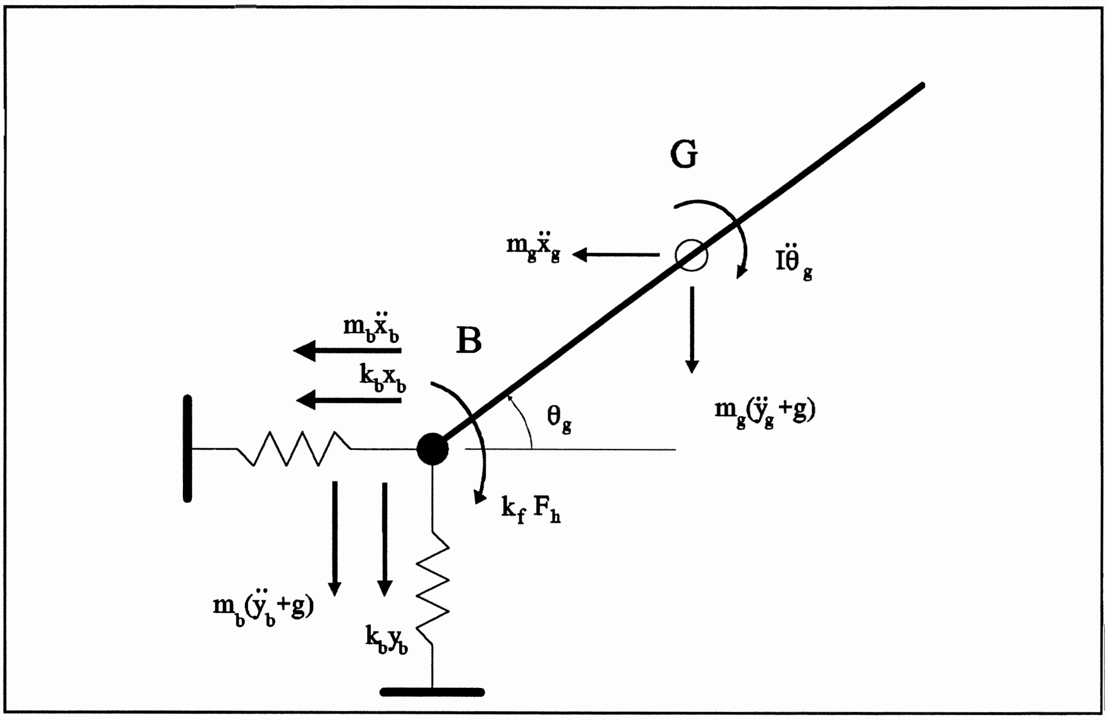

Figure 1. Diagrammatic model of gymnast performing giant circles.

Consider horizontal, vertical and rotational equilibrium

$$\left.\begin{aligned}
&-k_b x_b - m_b\ddot{x}_b - m_g\ddot{x}_g = 0\\
&-k_b y_b - m_b(\ddot{y}_b+g) - m_g(\ddot{y}_g+g) = 0\\
&-I_g\ddot{\theta}_g + m_g\ddot{x}_g l_g\sin\theta_g - m_g(\ddot{y}_g+g)l_g\cos\theta_g\\
&\quad - sign(\dot{\theta})\ k_f\left|k_b x_g\cos\theta_g + k_b y_g\sin\theta_g\right| = 0
\end{aligned}\right\} \tag{2}$$

The horizontal and vertical acceleration of the gymnast can be redefined in terms of the generalised variables. If the point B is the origin of a set of orthogonal axes then the displacement of any point, G, on the link is given by

$$\hat{R} = \hat{R}_0 + \hat{r} \tag{3}$$

where $\hat{R}_0$ is the position vector of the point B measured from the inertial reference frame, $\hat{R}$ is the position vector of the point G measured from the inertial reference frame, and $\hat{r}$ is the position vector from B to G.

For the case where G and B are points on the same rigid body (Chester [2])

$$\frac{d^2\hat{R}}{dt^2} = \frac{d^2\hat{R}_0}{dt^2} + \ddot{\theta}\times\hat{r} + \dot{\theta}\times(\dot{\theta}\times\hat{r}) \tag{4}$$

The last two terms in (4) correspond to the effect of the angular acceleration caused by the rotation of the local reference frame and the acceleration component introduced by the angular velocity of the local frame.

Combining (2) and (4) and writing in matrix form gives a system of equations in the form

$$[A]\{\ddot{x}\} + \{C\} + \{K\} = \{F\} \tag{5}$$

where

$$[A] = \begin{bmatrix} m_b + m_g & 0 & -m_g l_g \sin\theta_g \\ 0 & m_b + m_g & m_g l_g \cos\theta_g \\ m_g l_g \sin\theta_g & -m_g l_g \cos\theta_g & -m_g l_g^2 - I_g \end{bmatrix}$$

$$[C] = \begin{bmatrix} -m_g l_g \cos\theta_g \dot{\theta}_g^2 \\ -m_g l_g \sin\theta_g \dot{\theta}_g^2 \\ 0 \end{bmatrix}$$

$$[K] = \begin{bmatrix} -k_b x_b \\ -k_b y_b \\ -sign(\dot{\theta}_g)\ k_f\ |k_b x_b \cos\theta_g + k_b y_b \sin\theta_g| \end{bmatrix}$$

$$[F] = \begin{bmatrix} 0 \\ -(m_g + m_b)g \\ m_g g l_g \cos\theta_g \end{bmatrix} \qquad [x] = \begin{bmatrix} x_b \\ y_b \\ \theta_g \end{bmatrix}$$

The time history of displacements, velocities and accelerations can be obtained by solving the system of equations given by (5). These equations are non-linear, differential equations and their solution requires some form of numerical integration procedure.

Instead of trying to satisfy the equations of motion at any time t, it is aimed to satisfy these equations at discrete time intervals, Δt apart. The derivatives appearing in the equations of motion are approximated and a step by step solution using time steps, Δt, is obtained. Special advantage is taken of the form of the equations of motion and the average acceleration between time steps is assumed to be constant (Bathe [1]). This is an implicit integration scheme and iteration is required at each time step. Gatto [3] found that this scheme required one quarter of the computer CPU time compared to a Runge Kutta Order 4 numerical integration scheme. Figure 2 presents a flowchart of the solution algorithm that is used and convergence was rapid requiring at most three iterations per time step.

For each time step

Assume $\{\ddot{x}\}^{(0)}_{t+\Delta t} = \{\ddot{x}\}_t$

For $i = 1$ *to maximum number of trials*

Calculate $\{\dot{x}\}^{(i)}_{t+\Delta t}$, $\{x\}^{(i)}_{t+\Delta t}$ *using constant average acceleration*

Formulate $[A]$, $\{C\}$, $\{K\}$ *and* $\{F\}$

$$\{\ddot{x}\}^{(i+1)}_{t+\Delta t} = [A]^{-1}\left(\{F\} - \{C\} - \{K\}\right)$$

If $\{\ddot{x}\}^{(i+1)}_{t+\Delta t} - \{\ddot{x}\}^{(i)}_{t+\Delta t} <$ *tolerance*

Then goto next time step

Next i

Next time step

Figure 2. Flow chart for solution of non-linear equations.

3. Examples

Representative values of the mass and inertia of the gymnast are m_g = 77.3 kg, I_g = 13.54 kg m^2 and r_g = 1.194 m (McConville et al. [6]). The properties of the high bar are d = 28.8 mm, m_b = 4.7 kg and k_b = 5.247 x 10^4 N/m. The dynamic coefficient of friction is taken to be k_f = 0.01 N-m/N. The initial conditions are with the gymnast at rest and θ_g = 85 deg. For this analysis a time step of 0.0001 secs and a tolerance of 10^{-6} were required for convergence.

Figure 3 shows a plot of the resultant force at the hands of the gymnast versus time. The maximum force is 4.33 BW (body weight) and this occurs when the angle of rotation, θ_g, is at -89 degrees, i.e. nearly vertically down. The deflection of the bar at this point is 63 mm vertically and 0.1 mm horizontally. Figure 3 also shows the predicted force versus time curves when using different values of the dynamic coefficient of friction. As can be seen, there is very little difference between the predicted forces (about 2 percent difference between the peak forces). The maximum torque generated is 33 N-m.

The maximum force at the hands predicted by a single degree of freedom model (i.e. a model where the bar is considered rigid) is 4.56 BW. When the stiffness of the high bar is included in the model, the predicted force is 4.33 BW, a decrease of 5 percent. Figure 4 shows the predicted force at the hands versus time for three values of bar stiffness. Decreasing the length of the bar from 1.84 m to 1.46 m or increasing the bar diameter from 28.8 mm to 36.3 mm doubles the stiffness of the high bar. As can be seen from figure 4, modifying the stiffness of the high bar makes very little difference to the values of the predicted force - doubling the stiffness of the bar increases the peak force from 4.33 to 4.39 BW.

Based on this analysis, it can be seen that modifying the stiffness of the bar or reducing the friction between the gymnast's hands and the bar makes very little difference to the values of the maximum forces. Kopp and Reid [4] measured the

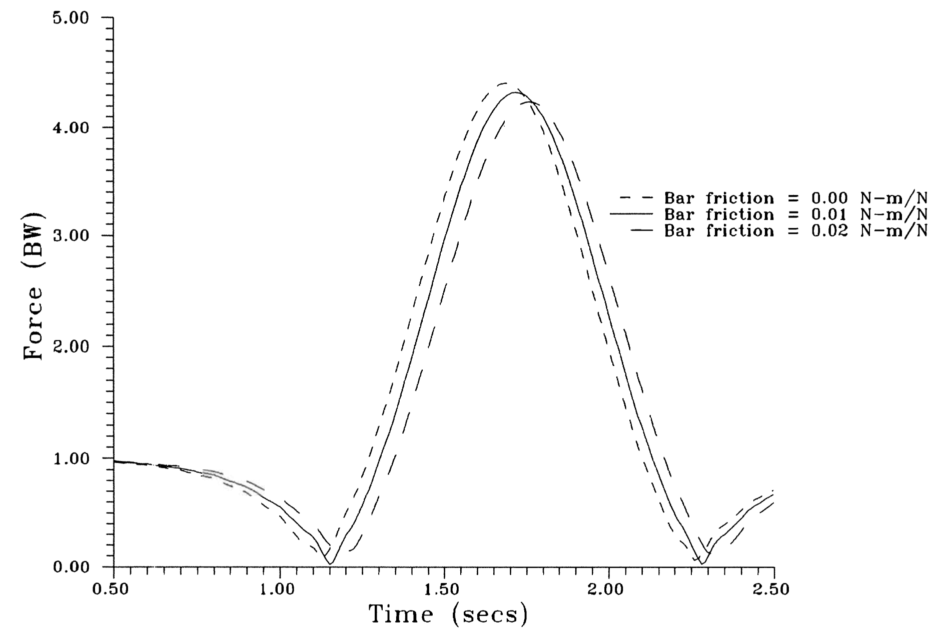

Figure 3. Resultant force at the gymnast's hands for varying values of dynamic friction.

maximum force at the gymnast's hands and found it to be between 3.45 and 3.7 BW which is 15 percent smaller than the predicted values from this model. However, the values of the maximum torque are within the same range.

4. Conclusion

A mathematical model of a gymnast performing giant circles on the high bar has been presented. This model uses numerical integration to solve the nonlinear equations of motion and obtain the time history of displacements, velocities and accelerations. The following conclusions are presented:

1. The maximum forces at the gymnast's hands are approximately 4.3 BW.
2. The effects of the bar stiffness and the friction between the hands and the bar on the maximum forces are very small.

This model is part of a study which is investigating the forces on the wrists of gymnasts. Results from the experimental study will be combined with the mathematical model and used to obtain a better understanding of the mechanical principles involved in the proper execution of the giant swing.

Acknowledgments

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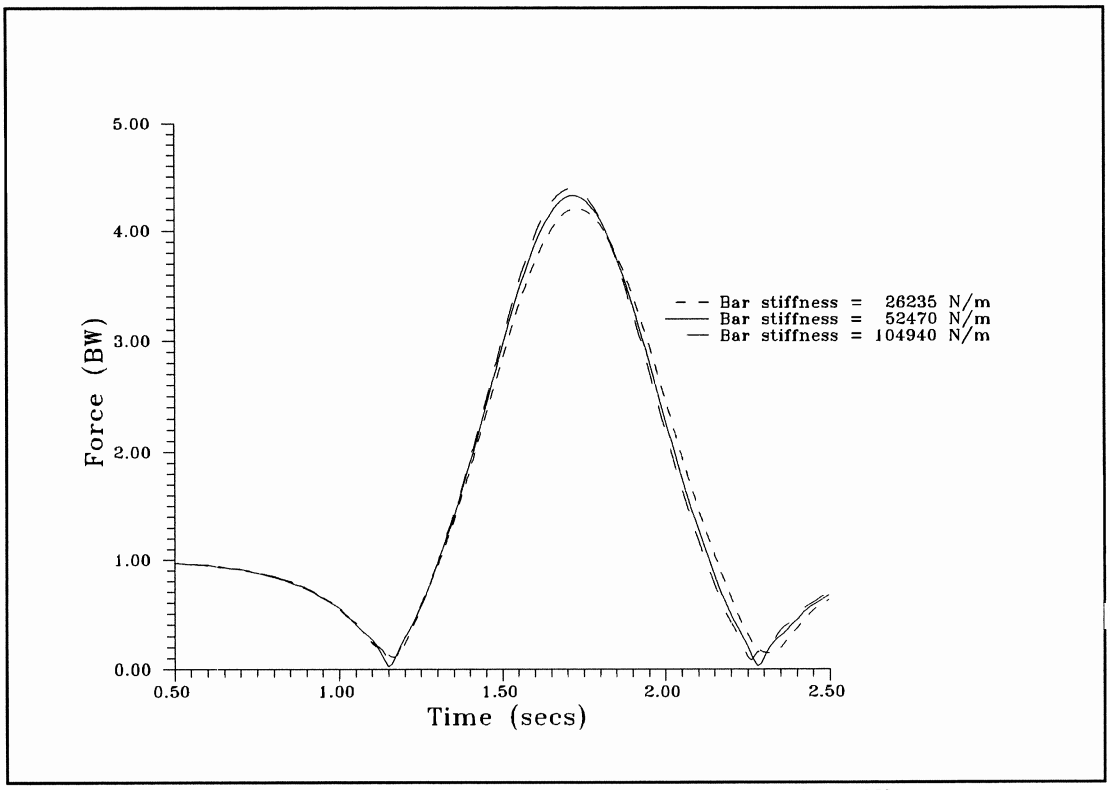

Figure 4. Resultant force at the gymnast's hands for varying values of bar stiffness.

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A TAXONOMY OF SPORTING EVENTS APPLYING OPERATIONS RESEARCH METHODOLOGY

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Abstract

This paper considers the application of mathematical and OR methodologies that have been used to analyse sporting activities. By considering the specific techniques used, a structure has been developed which is applicable to the sporting activity process. Finally there is consideration as to why these techniques are not more widely utilized by sports organizations.

Introduction

This year being an Olympic Games year, there has been considerable exposure to the help being provided by science to our sporting participants. As pertaining to the television media this has involved biomedicine and biomechanics provided by the facilities incorporated in the Australian Institute of Sport (AIS), primarily by the Sports Science Institute and the Biomechanics Department.

Unfortunately these references do not mention the disciplines of mathematics, statistics or operations research. It is not that these disciplines are not being used to assist in improving the performance of the athletes. However the techniques employed assist the scientists to perform their analysis, but unfortunately there is no perception of these disciplines being used to assist athletes.

This conference must therefore address two questions :

a) Where can mathematics, statistics or operations research help to improve the performance of sportspersons, the conduct of sporting events, or the enjoyment by participants and spectators?

b) Why are existing techniques applicable to the sports arena in the areas of mathematics, statistics or operations research not being implemented?

History

Sport has been an integral component of the social framework for thousands of years. In fact in the context of the Olympics, the modern variety pale into insignificance with the ancient Olympics, which were held for over one thousand years from 776BC to 394AD. However in the early times the achievements of participants was for the honour it bestowed on winners in the eyes of their compatriots. Sport as it is now perceived only had its genesis in the middle of last century, when a number of disciplines, mainly cricket and baseball, were organised into formal competitions. From these times comprehensive statistics were collected which were then compiled into documents which ranked participants in a manner amenable to players, commentators and spectators. As an example in the game of cricket, Wisden has been published since 1863, and the statistics which are ranked have changed very little until the advent of one day cricket, which while still producing many of the measures associated with

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test and county cricket, generated a new set such as run rate for batsmen and economy rate for bowlers.

Very little analysis was performed on the wealth of data in a variety of fields until the late 1950's and early 1960's. This could be attributed to a number of factors, including the availability of computers, and the fact that in the field of operations research, which had its birth in Britain during the Second World War, many of the techniques were still being developed and formalised. As well in the years after the war, major effort was devoted to economic reconstruction, with scientists dedicated to assisting in this field. It can be argued that it was not until the Eastern Bloc decided to use sport, especially in the Olympic Games, as a means of measuring the "power" of a nation that sport was perceived as an acceptable area for scientific research and application.

In the late 1970's two books were published as a joint project, one by Machol, Ladany and Morrison [56], and a second by Ladany and Machol [55], which attempted to bring together the application of quantitative methods and systems analysis that had previously been applied to sport.

The first of these books contains fifteen articles written for professionals in operations research. The techniques used can be categorised as follows :

Probability	6
Heuristic	3
Markov Process	2
Dynamic Programming	1
Network	1
Simulation	1
No technique	1

The second contains thirty four articles written for the "educated layman". However it is interesting to note that this book is prefixed by an article "A Word to the Non-Mathematician" which contains references to the skew of a distribution, Gaussian distributions, error functions, Bernoulli distributions, regression analysis, derivatives and integrals, transient Markov matrices, and conditional probability. The editors further highlight the problem with the statement "... uses heavy mathematics (vector calculus) which the layman can skip to get to his conclusions ...". Is it no wonder these techniques were not embraced by sports coaches and practitioners.

The editors categorise the articles by the techniques used and application as follows :

	Baseball	Football	Track & Field	Miscellaneous
Probability and Statistics	12	5	2	7
Matrix Algebra	1			
Calculus			2	1
Optimization Theory	3		2	1
Computers	8	2	1	2

Care however is needed in interpreting the above figures, since some authors have multiple references, while some articles are listed incorrectly with respect to OR techniques (eg a number listed as Probability and Statistics involve models solvable as Markov Processes). Of interest are some of the conclusions reached :

a) "... decisions which were routinely made ... and which are being routinely made by today's managers, are obviously wrong -- obvious, that is, to any reader of this book." While not stating it explicitly, there is the suggestion that managers in the future will be required to implement the results of the analyses included. Unfortunately a number of the authors give reasons why this may not be the case. These reasons are invariably tied to the assumptions that are required to model the sport. In particular they invariably ignore the psychological factors that managers regard as important. Bellman [54] sums this up by stating "As in poker, one can do very well playing percentages; but, if one wants to win big, one has to play psychology".

b) "Sometimes strategies are not adopted for reasons unconnected with winning the competition. ... managers who end up looking stupid are not likely to keep their jobs."

c) "Sometimes strategies are not adopted out of sheer conservatism. The Australians introduced a new way of placing oars in an eight-oar shell in the early 1960's, ... , but in the 1976 Olympics nobody except the Australians had adopted this obvious improvement. Perhaps it will have to wait until the Australians win an international race, and then everyone will copy them."

The last conclusion was prophetic in that Australia did eventually win in international competition, and other nations did redesign their eight-oar shells. However Townend [57] discusses these changes in his book, but credits the Australian design to the Italians, even though he references the article by Brearley which appeared in Ladany and Machol [55].

Scope of Analysis

Ladany and Mahol attempted, as an appendix to their book, to provide an Annotated Bibliography and a comprehensive Reference for articles on sport written before 1977. No attempt has been made since then to reproduce this. It thus seemed appropriate to use, as a base for the analysis, all articles published since this time, reproducing a comprehensive reference. Since the breadth of articles has become substantial, along with the journals within which they can be published, it was necessary to constrain the search. This was achieved by primarily restricting articles to those published in English language Operations Research journals exhibiting an interest in publishing articles relating to applications in the sports domain. The list searched was :

Journal of the Operations Research Society
European Journal of Operations Research
Operations Research
Operations Research Letters
Interfaces
Computers and Operations Research
Management Science
Siam Review
Naval Research Logistics

For those articles within these journals, referenced articles were also considered where it was considered that they would augment the list. Thus the list also includes articles from :

Journal of the Royal Statistics Society (Series C)
Australian Mathematical Society (Series B)
Australian Journal of Statistics
New Scientist
Conferences of the Australian Society for OR

In searching for referenced articles it became apparent that not only did the inclusion of sports related articles depend on the policy of the journal, but also on the editor. As well, and indicative of the problem to be addressed, the average number of published articles in the 15 years covered was 3 per year.

Classification

In attempting to classify the bibliography there are a number of criteria that can be employed. The obvious criterion is the sporting area that is addressed. The following table gives the number of articles in each of the sporting areas :

Athletics	5
Baseball	1
Basketball	3
Board Games	3
Cricket	3
Darts	2
Diving	1
Dressage	1
General	10
Gymnastics	4
Ice Hockey	4
Orienteering	6
Racquet Sports	7
Soccer	1
Swimming	1

A number of observations should be made before any conclusions are drawn from the above classification. Firstly, some of the figures are deceptive in that an article, particularly in ice hockey and orienteering, has generated a succession of articles updating or correcting theories presented in earlier articles. Secondly, the popularity of a sport does not necessarily correlate with the number of articles. In particular soccer generates one article, while orienteering generates six. It may be argued that this is indicative of the relative age of the two sports. However in Ladany and Mahol's bibliography, soccer is mentioned only twice, and orienteering not at all, while less popular sports are have many more references.

Articles may also be classified by the analytic technique primarily used to address the problem. In order of popularity have :

Probability	16
Heuristics	12
Linear Programming (including Integer Programming)	6
Dynamic Programming	6
Simulation	3
Analytic Hierarchical Process	2
Assignment	2
Travelling Salesman	2
Graph Theory	2

In considering these techniques from the view of those involved in the sporting process, either analysts, coaches, participants or spectators, it is unlikely that many would have sufficient understanding of any of the techniques to be able to address the results with confidence. As will be discussed later, this is the major barrier to cooperation between those involved in methodologies and participation.

Analysing the methods deployed, it is possible to break to complete process of a sport into a number of discrete components.

i) Prior to the staging of the event

ii) Strategies to be deployed during the event

iii) Event management

iv) Changes that would affect the outcome of the event

v) Post event analysis

These components can be seen to impinge on sport in a number of different ways. Thus i), iii) and iv) are involved with techniques which lead to more efficient conduct of a sport, ii) involves improving the performance of the competitor, while v) is for the enjoyment of participants and spectators.

Combining the above classification with the methodologies used produces the following matrix:

	i)	ii)	iii)	iv)	v)
Probability		9		5	2
Heuristics	3	5	1	1	2
Linear Programming (including Integer Programming)	5				1
Dynamic Programming		5		1	
Simulation		1	2		
Analytic Hierarchical Process					2
Assignment	1			1	
Travelling Salesman	2				
Graph Theory					2

The previous table highlights the fact that certain techniques are more suited to being applied to problems associated with different components of the sporting process.

i) Prior to the staging of the event : These are generally concerned with allocating participants to events [24][13][31][11][23], umpires to events [52][53][15], scheduling events [4][43], and to optimizing travel schedules [5]. Since there is no time pressure in determining appropriate results, these techniques may provide the greatest benefit to sporting organisations. In fact many of the articles represent descriptions of successful applications. However since they have been tasked for specific problems, the general application of the algorithms deployed is limited.

ii) Strategies to be deployed during the event : These articles are best classified by the particular sport, involving a coach's decision on pulling a goalie in ice hockey [36][14][39][49], when and where to serve in racket sports [3][51][44][38], the design of optimal routes in orienteering [47][20][21][27][42][25], cricket rates [7], doubling in backgammon [48], where to aim in the long jump [30], where to aim in darts [29], where to throw from in a basketball shootout [18], and pole-vault strategies [26]. The applications in this group fall into two subcategories. The first is those techniques which can be practically applied, such as when to pull the goalie, doubling in backgammon, where to aim in the long jump, and where to aim at darts. Except for backgammon, these are factors which the coach can control, such that successful application improves the chances of the competing unit, without imposing any extra burden on the unit. The remainder fall in the second subcategory, where application is inappropriate, generally because there is insufficient time to carry out the required computation.

iii) Event management : These are concerned with improving the finish of mass participants in long distance races [33][16], and improving the methodology of splitting ties in Swiss type draws [40]. As with the first group, the methodologies employed can be determined prior to the start of the event. However unlike the first group, these methods are generally employable to a variety of events having similar characteristics.

iv) Changes that would affect the outcome of the event : These involve modifying the rules of existing games [1][17][32][12], and computation of probabilities of certain results [41][28][34][37]. The analyses in this group are for interest only, with little practical application, except for the analysis of events with subjective judging, where the application of new methods may lead to fairer results.

v) Post event analysis : All the articles with this classification involve methods of ranking the results of competitions [50][35][2][8][6] [22][45][19][9][46]. The articles in this group are for commentator and spectator interest only, with the exception of the first article which deals with the bias associated with events where competitors are divided into two or more divisions. It is interesting to note that the national rugby league management took this retrograde step some years ago. This highlights the fact that in any analysis of a particular sporting activity, the conclusions reached may not be valid unless all factors are included in the analysis, a situation that may be impossible since some factors may not be capable of modelling, or the resultant model is unable to be solved.

Conclusion

This paper has attempted to classify articles related to sport published in OR journals, both by the techniques used and the application area within the sporting process. While many of the techniques considered are academic in their analysis of the particular sport considered, offering little to the particular sport's enhancement, many others have addressed areas which could lead to improvements, both in sports performance and in organizational efficiency. Unfortunately only a few of the latter applications have actually been implemented.

Generally those that have been implemented have been attributable to the fact that the sporting organization has perceived a problem, which has then been addressed, with the article being a bonus to the author. This should not be taken as indicating that progress can only be achieved if a need is perceived. Rather it should indicate that acceptance of any advance generally requires interaction between academic and sports organization.

This suggests the second observation about the sports environment, which is that there are many psychological and environmental factors involved. These are often excluded in OR models, yet are often regarded as critical in sporting activities, especially by coaches who administer significant control over sporting units. Without the support of these people it is unlikely that any new advance has much chance of implementation.

The coaches are often people who have retired from the sport, having little training in statistical and OR techniques. This is not to suggest that they will not embrace complex scientific approaches. However much like the interaction between managers and scientists, if you wish to employ the scientists discipline, it is necessary to expose the decision makers to the advantages of scientific analysis, without confusing the decision makers. Only then can it be hoped that they will embrace the results of scientific analysis.

Thus, for there to be a significant increase in the use of scientific analysis within sport, it may be necessary for scientists to provide a simplified explanation of their discipline, and the advantages it might bring, rather than as is often done, produce complex methodology, and wait for it to be implemented.

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HANG GLIDERS, WOBBLES AND MYTHS

Maurice N. Brearley[1]

Abstract

Hang gliders are usually called 'Weight-Shift' gliders because of a belief that manoeuvres are initiated by the pilot shifting weight to appropriate positions beneath the canopy or wing which supports him. Theoretical and experimental evidence will be produced to show that this belief is pure myth, and that hang gliders would be more appropriately described as 'Weight-Non-Shift' gliders.

A popular hang glider manoeuvre when soaring in front of a ridge is the 360° turn. The path described relative to the ground when making such a turn in a uniform wind will be determined analytically. Graphical representation of the results will be presented, showing clearly the danger inherent in such a turn.

Euler's equations of motion of a rigid body will be used to investigate stability of motion in various sporting situations such as the flight of a discus, the motion of a lawn bowl, and the behaviour of a Rugby or Australian Rules football. Evidence will be produced to refute a myth which is given much credence in lawn bowls communities.

1. The Dynamics of Hang Glider Manoeuvres

Hang gliders are usually called 'Weight-Shift' gliders because of a belief that manoeuvres are initiated by the pilot shifting his weight to a new position by means of his arms. Hang gliders can, however, be more

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accurately described as 'Weight-Non-Shift' gliders, since the actual shift of the pilot's weight during a conventional manoeuvre is very small.

When a pilot is in straight and level flight as shown in Figure 1, his weight W acts in the same straight line as the resultant R of the lift and drag forces, but in the opposite direction.

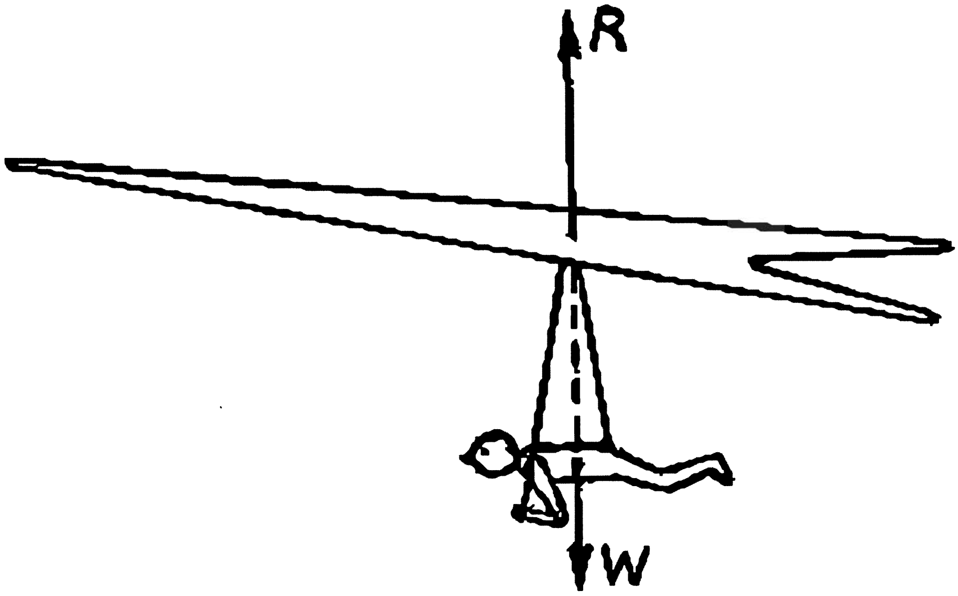


Figure 1. Hang glider in straight and level flight

In Figure 1, the glider is in equilibrium under the action of the opposing forces W and R.

Now consider the case when the angle of attack of the wing is increased in order to gain altitude. The usual (but incorrect) explanation of the process is as follows:

The pilot pushes himself back from the control bar of the A-frame, causing his weight W to be no longer in the same line as the resultant aerodynamic force R, as shown in Figure 2.

In Figure 2, the two forces W and R form what is known as a couple. This has a moment C, depicted in Figure 2 as a rotating influence on the glider. The glider is rotated by the couple into the nose-up attitude shown in Figure 3.

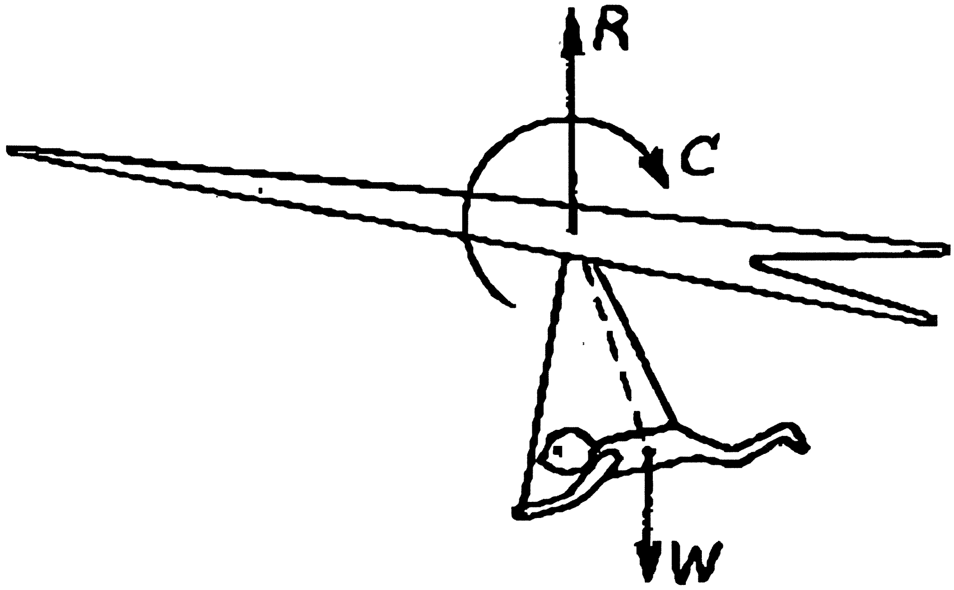


Figure 2. The usual picture of the first stage in an increase in angle of attack of the wing.

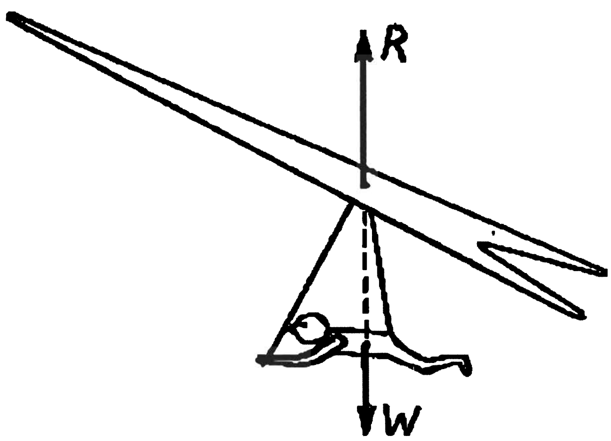


Figure 3. The final stage in the increase in angle of attack of the wing.

In Figure 3, the forces W and R are again in the same straight line, and the rotating moment C has reduced to zero. (The force R will, in general, have increased in value, causing the glider to start to gain altitude.)

Similar explanations can be given for other manoeuvres, such as adopting a nose-down attitude or producing turns to left or right.

Though plausible, explanations like the foregoing, which are based on the concept of weight-shift, are far from the truth of the matter. In the usual situation in which the mass of the pilot greatly exceeds that of the glider, a more accurate explanation of how the attitude in Figure 3 arises is as follows:

When the pilot pushes out on the control bar, the bar moves forward and the pilot moves back. The relative amounts of these two movements are governed by the relative magnitudes of the inertia of the glider and the inertia (or weight) of the pilot. It is easier to rotate the glider into a nose-up attitude than it is to move back the comparatively heavy pilot, and consequently glider rotation dwarfs pilot movement. This is why hang gliders are not weight-shift gliders to any significant degree.

Exactly the same argument applies in the case of a turn to left or right. The inertia of the glider about the axis of roll is, however, greater than about the axis of pitch, and so it is not as easy to make the glider bank as it is to make it nose-up or nose-down. In turn, therefore, pilot weight-shift would be slightly more significant, but still not nearly enough for it to be regarded as the main influence.

A more accurate view of a hang glider manoeuvre is to regard the pilot and his harness as providing a nearly vertical plumb line, relative to which the canopy of the glider is orientated in the desired position by the pilot's manipulation of the A-frame. With this more accurate picture of glider operation, the initial stage of inducing a nose-up attitude would be as depicted in Figure 3, with Figure 2 irrelevant.

During violent manoeuvres the pilot may, of course, be swung far from a vertical line of support, and the foregoing arguments are not intended to cover such situations.

A precise mathematical analysis of a hang glider manoeuvre could be made. The results would support the theory advanced in the preceding paragraphs. In order to obtain some quantitative results it would be necessary to determine the principal moments of inertia of a typical hang glider. This could be done if the weights and sizes of all of its constituent members were available, together with a plan showing their relative locations when assembled.

An easier method of investigating the behaviour of a hang glider is to build a scale model in which the action of a mutual force between the pilot and the A-frame can be suitably introduced. The writer has built such a model, in which the weight of the pilot is about three times that of the glider. The model is suspended in equilibrium by two strings in such a way as not to impede an increase in angle of attack of the wing. A separation of the pilot and the A-frame is produced pneumatically by pumping air along a thin flexible tube to a balloon between these two members. There is virtually no movement of the mass representing the pilot as the angle of attack of the wing increases. The model thus substantiates the view that the pilot is acting as an almost motionless plumb line with respect to which he alters the angle of the wing by movement of the A-frame.

The model could be modified to investigate its behaviour in other manoeuvres, such as turns. The results would be the same, virtual non-weight-shift of the pilot being exhibited in every case.

2. The 360° turn by a Hang Glider in a Uniform Wind

In 1969 I was asked a question by a pilot in the RAAF who had spent a lot of time on maritime work. He said:

"It is sometimes necessary for me to fly in a circular path, relative to the ground, around a stationary object such as a dinghy which is in the water below. If there is a wind blowing, I have to make continual changes in bank, turn and throttle setting to achieve this. Can you tell me the shape of the curve that I must be flying relative to the air in order to achieve the circular ground path?"

After much work I was able to answer his question. The shape of the curve depends on the ratio of the speed of the wind and the airspeed of the aircraft. The curve is similar to (but not identical with) those in Figures 5a and 6a.

The parametric equations of the curve involve an elliptic integral of the second kind. I coined the name 'aeroid' to denote the curve, and was rather proud of this, but nobody else ever used it. My published paper on the subject (Brearley [1]) caused a monumental lack of interest; nobody wrote to me and asked for a reprint of it. Even the ability of the theory to predict the increased fuel consumption due to a wind created no Air Force interest.

One of my students at the RAAF Academy was called Wayne Blackmore. He became a paraplegic in 1973 in a tow-glider accident at Point Cook. He later became the first President of the Hang Gliding Federation of Australia. He was also at one stage President of the South Australian Hang Gliding Association, and was Australia's first paraplegic hang glider pilot. His life story has been the subject of a book (Brearley [2]).

In 1975 Wayne said to me:

"One favourite manoeuvre in a hang glider is a 360° turn. If there is a strong wind blowing, you have to be careful that it doesn't carry you into a ridge you are soaring in front of. Do you know the shape of the flight path relative to the ground when a 360° turn is done at a constant rate in a uniform wind?"

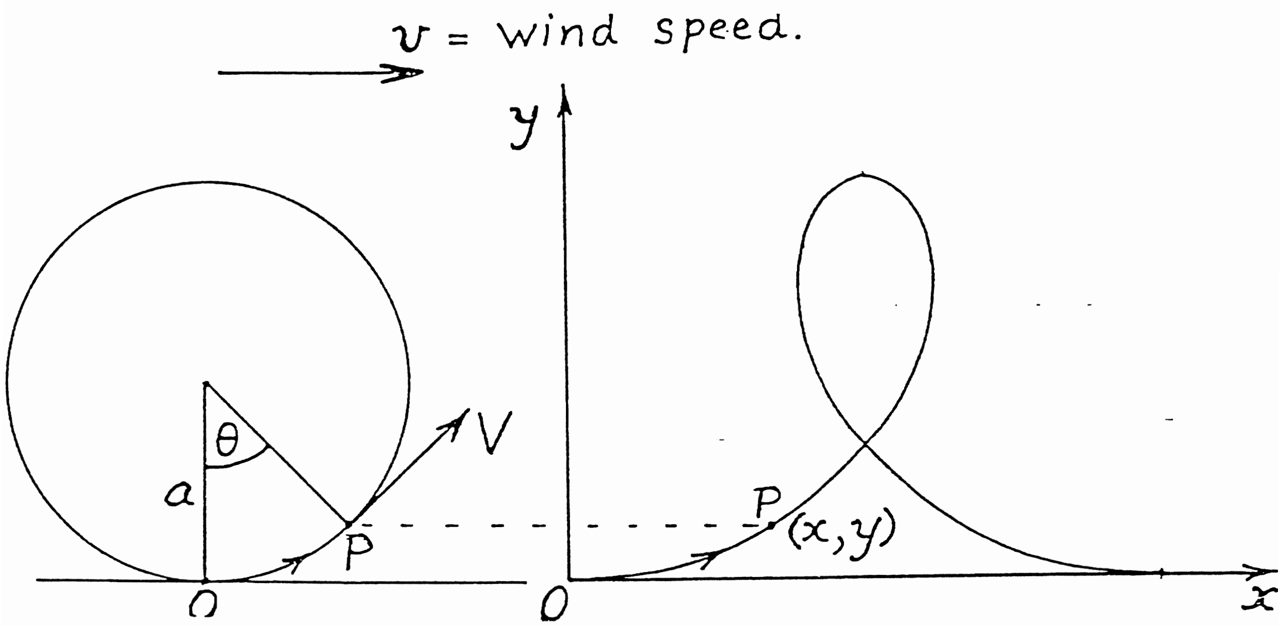
I replied at once:

"This is a problem I have already solved. The curve is an aeroid, and I wrote a paper about it."

It was obvious to me that the two situations were equivalent, and for Wayne's benefit I prepared to copy from my 1970 paper the relevant mathematics. This, however, is one of these intriguing cases where something that is 'obvious' is, in fact, false. The hang glider's ground path is not an aeroid, though its shape is very similar to one. The mathematics required to plot the glider's ground path is much simpler than that needed for an aeroid.

We will use the following notation:

v = wind speed (assumed constant),
V = glider airspeed (assumed constant),
k = v/V
a = radius of circular turn.



Figures 4a and 4b

If the angle θ of turn is performed in time t, it is easily seen that the coordinates of the ground path are

$$x = vt + a \sin \theta$$
$$y = a(1 - \cos \theta),$$

where the time t is found by integrating the equation

$$d\theta/dt = V/a = \text{constant}.$$

Hence $\theta = Vt/a$ and $t = a\theta/V$, giving the ground path equations

$$x = a(k\theta + \sin \theta),$$
$$y = a(1 - \cos \theta).$$

These equations enable the ground path to be plotted for any selected value of k. The path is not an aeroid; for k = 1 it is a cycloid; for other values of k it resembles a trochoid.

An example of the ground path is shown in Figures 5a and 5b for k = 1/2, which is the case of a light wind. The path in Figure 5b for a turn that begins and ends upwind is found by relocating in an obvious way the two halves of the path in Figure 5a. The arrows on the curves shown the headings of the glider at various stages of the turns.

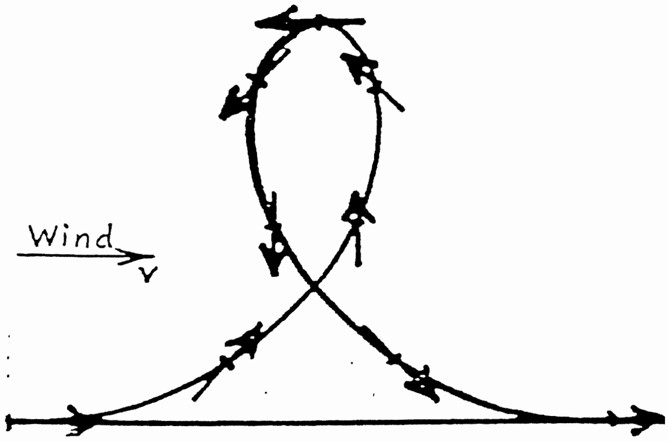


Figure 5a. Ground path of glider performing a constant rate 360° turn, for k = v/V = 1/2. The turn begins and ends downwind.

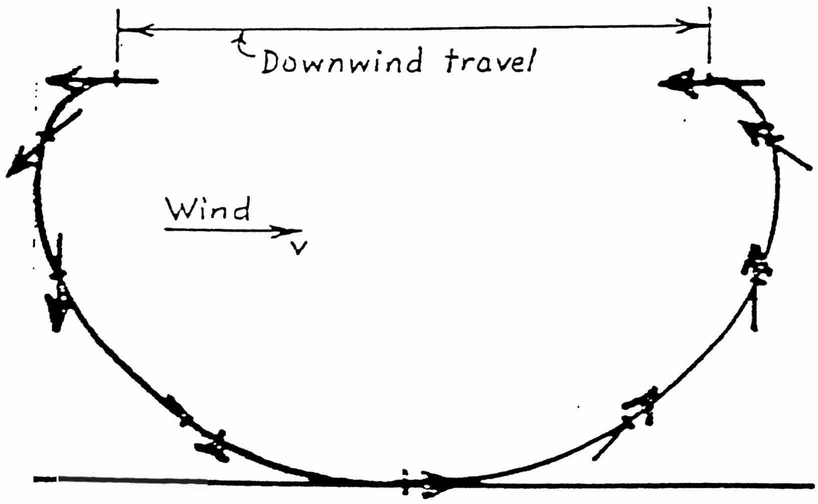


Figure 5b. As for Figure 5a, but turn begins and ends upwind.

Figures 6a and 6b show the ground path for k = 1, which is the case of a fairly strong wind.

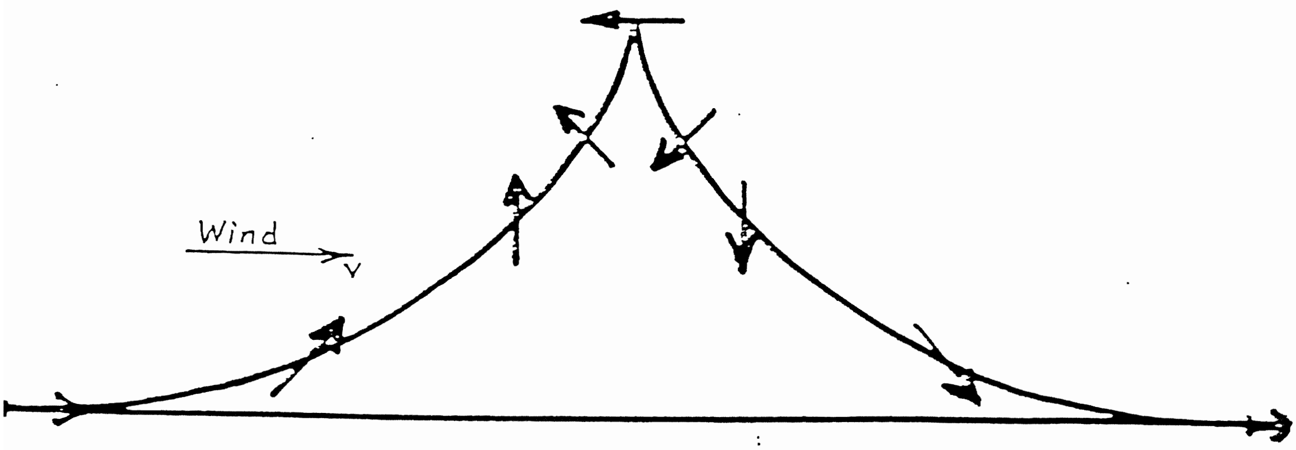


Figure 6a. Ground path of glider performing a constant rate 360° turn, for k = v/V = 1. The turn begins and ends downwind.

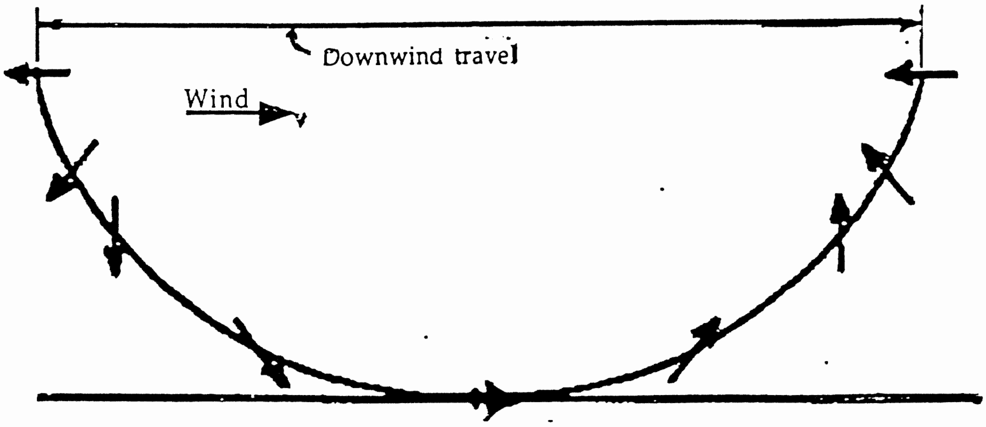


Figure 6b. As for Figure 6a, but turn begins and ends upwind.

The implications for hang gliding safety are obvious. For a glider soaring in front of a slope towards which a wind is blowing, a 360° turn is usually begun when heading upwind, for which Figures 5b and 6b are the relevant ones.

As a numerical example, suppose that the radius of turn of the glider relative to the air is 17 metres. Then by scaling off Figures 5 and 6 it is found that the downwind travel during a complete turn is 53 metres if the wind speed is half the glider airspeed, and is 110 metres if these speeds are equal.

Experienced glider pilots are very aware of the danger of being swept into a slope while performing a 360° turn. For less experienced pilots, the visual evidence of the downwind travel component in Figures 5b and 6b may be a useful warning.

3. Euler's equations of motion for a rigid body

If the external forces on a rigid body have zero (or negligible) moment about the centre of mass G of the body, then Euler's equations about principal axes of inertia at G are

$$A\dot{\omega}_x + (C - B)\,\omega_y\omega_z = 0 \qquad (1)$$

$$B\dot{\omega}_y + (A - C)\,\omega_z\omega_x = 0 \qquad (2)$$

$$C\dot{\omega}_z + (B - A)\,\omega_x\omega_y = 0 \tag{3}$$

Here A, B, C are principal moments of inertia at G, and ω_x, ω_y, ω_z are the components about axes Gx, Gy, Gz of the angular velocity of the body (and of the axes, since the axes are supposed to be fixed in the body and moving with it). The axes with respect to which ω_x, ω_y, ω_z are measured are fixed axes which at any instant are instantaneously coincident with the moving axes.

Euler's equations apply also in the case of a rigid body rotating about a fixed point O in the absence of any external moment about O. The axes concerned are then principal axes of inertia at O.

4. The Case of Kinetic Symmetry

This means that just two of the principal moments of inertia are equal, for example

$$A = B \neq C$$

Then the body has one axis of symmetry, namely Gz. Examples are a discus, a lawn bowl, a baseball bat, and (to a good approximation) a Rugby or Australian Rules football.

In this case equation (3) shows at once that

$$wz = \text{constant} = K, \text{ say.} \tag{4}$$

The Euler equations (1) and (2) then become

$$A\dot{\omega}_x + (C - A)K\omega_y = 0 \tag{5}$$

$$A\dot{\omega}_y + (A - C)\,K\omega_x = 0 \tag{6}$$

Differentiating (5) with respect to t, and substituting from (6) for the resulting value of $\dot{\omega}_y$, we are led to

$$\ddot{\omega}_x + [(C - A)^2K^2/A^2]\omega_x = 0$$

or

$$\ddot{\omega}_x + n^2\omega_x = 0 \tag{7}$$

where

$$n = |C - A|\,K/A \tag{8}$$

The solution of (7) is

$$\omega_x = K_1 \cos nt \tag{9}$$

where K_1 is constant, if we take

$$\omega_x = K_1 \text{ at } t = 0 \tag{10}$$

Equations (9) and (5) then show that

$$\omega_y = K_1 \sin nt \tag{11}$$

5. The Case of Rotation about the Axis of Symmetry Gz

Suppose the body is rotating about Gz with an angular velocity which is not small, and that it is then given (by some external agency) small components ω_x and/or ω_y of angular velocity.

Then K_1 in the condition (10) is small, and (9) and (11) show that ω_x and ω_y remain small and simply oscillate sinusoidally about zero values. The rotation about Gz is therefore stable.

We now consider some practical examples of this situation.

(a) The flight of a discus

A discus clearly has kinetic symmetry. When it is thrown, a discus usually has a significant angular velocity about its axis of symmetry, and often a small wobble is noticeable. Such a wobble is represented by equations (9) and (11). It persists throughout the flight because the air provides negligible damping moment.

(b) The behaviour of a lawn bowl

A bowl has kinetic symmetry, with $A = B < C$ if Gz is the axis of symmetry.

When launched in a slightly tilted or yawed position, a bowl has a small initial wobble as soon as it starts to roll, i.e. ω_x and ω_y are small and vary sinusoidally in accordance with equations (9) and (11). The wobble is very obvious because of the rotational motion of the central spot on either side of the bowl.

This wobble soon decays to zero, leaving the bowl running truly, in apparent contradiction of the earlier results. The reason is that a bowl is not in free rotation; the grass provides a moment of external force about G, and it is a part of this (proportional to the angular velocity) which damps the wobble (Brearley [3]).

One of the myths of bowling folk-lore is that of the 'canted bowl'. Many bowlers believe that if a bowl is canted (or tilted) on delivery, this tilt will persist during the travel of the bowl and thus alter its end position. In fact, any initial cant merely induces a wobble and does not result in a continuing steady tilt of the bowl.

(c) A Rugby or Australian Rules football

Apart from the small discrepancy resulting from the presence of the bladder valve and the leather laces outside it, the long axis (Gz say) is an axis of symmetry of the ball. If the ball possesses a significant spin about this axis, and also small initial angular velocity components ω_x and ω_y, the latter two components will vary sinusoidally in accordance with the foregoing theory. This wobble about the long axis will persist with very little amplitude reduction during the flight of the ball, as the resistance of the air is small enough for the motion to be virtually 'free'.

6. Stability of a Football after a Place Kick or Drop Punt

For a Rugby or Australian Rules football we have kinetic symmetry, with

$$A = B > C.$$

In a place kick or drop punt, the ball is given a fairly rapid rotation about an axis, Gy say, which is perpendicular to the long axis Gz. We want to investigate the stability of the subsequent motion of the ball.

Suppose that initially

$$\omega_y = K_2 \text{ at } t = 0, \qquad (12)$$

where K_2 is not small.

To investigate stability we suppose that ω_x, ω_z are given small values by some external agency, and we determine their subsequent behaviour.

Euler's equation (3) leads as before to

$$\omega_z = \text{constant} = K \qquad (4)$$

where K is now small; and as before, equations (1) and (2) take the forms (5) and (6) deduced earlier.

Also as before we deduce that

$$\omega_x = K_1 \cos nt \tag{9}$$

if we again take

$$\omega_x = K_1 \text{ at } t = 0, \tag{10}$$

where K_1 is small, and where

$$n = (A - C)K/A \tag{8a}$$

since $A > C$ (and we need not use $|C - A|$ as in (8)).

Substituting from (9) into (6) yields

$$\dot{\omega}_y = -nK_1 \cos nt,$$

therefore

$$\omega_y = -K_1 \sin nt + \text{constant},$$

$$= K_2 - K_1 \sin nt \tag{13}$$

in virtue of the initial condition (12).

Since ω_x and ω_z remain small and ω_y remains close to its initial value K_2, the rapid rotation about Gy is stable and persists during the flight of the ball.

7. The Case of Spherical Symmetry

This means that A = B = C, and there is no preferred axis, e.g. for a soccer ball, basketball, or tennis ball.

Euler's equations show that

$$\omega_x = \omega_y = \omega_z = \text{constant}$$

So if such a ball is given any sort of angular velocity it will persist throughout the flight of the ball, assuming of course that air resistance exerts negligible moment on the ball.

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FLIGHT OF A FOOTBALL

Geoff Aldis, Glenn Fulford, Rodney Weber[1]
and Neville de Mestre[2]

Abstract

The trajectory of a kicked football is modelled as a projectile under the influence of gravity and drag forces. Experimental data from measured 31 kicks is separated into two groups, a control group of 3 kicks and a test group of 28 kicks. The values for the drag coefficient are determined from the control group, and used with the test group to compare predicted and measured kicks. Only 8 of the 31 kicks fall outside the success criterion of less than 10% difference between predicted and measured distances obtained.

1. Introduction

Surprisingly little scientific work has been done on the football kicking problem. Daish [5] prepared a ballistic table for a soccer ball, kicked without spin, which included the effects of drag. He also gave a rough comparison between the ranges for two types of kick with a rugby ball, one in which the longitudinal axis of symmetry remains closely aligned with the tangent to the trajectory and the other for a ball tumbling end-over-end

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during flight. Brancazio [2] studied the relationship of kick length and time of flight (hang time) to the initial speed and launch angle of the ball by analyzing videotaped U.S. National Football League games. In a later paper Brancazio [3] gave an analytical description of the rotational dynamics of a spinning football.

The biomechanics of Australian Rules football kicking was investigated by Macmillan [9]. Then Cunningham and Dowell [4] reported experiments in which the distances of various kicks of an American football were compared for different initial launch speeds and angles. Cunningham and Dowell [4] classified the trajectories associated with these ellipsoidal footballs into three types, according to orientation throughout the flight. Their calculation of the distance to the first bounce (range) was based upon a re–scaling of the gravity–only solution for the range. A dynamically correct description must be based, however, upon the equations of motion, including the effects of a drag force.

A complete theoretical analysis of kicking a football should concentrate on three separate parts:

(i) the properties of the ball being kicked

(ii) the biomechanics of the kicking action *and*

(iii) the aerodynamics of the ball plus the effect of gravity during its flight.

For the first, the shape of the ball and its relevant dimensions need to be specified. Also its mass and drag coefficient need to be measured. A number of different footballs are commonly used (American Football, Rugby Union, Australian Rules, Rugby League and Soccer). Of these only the soccer ball is approximately spherical, the others being approximately ellipsoidal.

The biomechanics of the kicking action can be investigated to determine the initial linear speed, initial angular speed and angle of projection of the football. Alternatively these measurements can be obtained from films or videos of the kick.

Finally the aerodynamics of the football needs to be understood so that models that include drag, lift or sideways forces can be tested. In this paper the main model presented contains both gravity and drag forces, and a brief mention of lift is included, but not in great detail.

Name	Location	Year	Type of Kick	Distance (m)
F.C. Cooper	Fremantle	1895	Place	83.5
D. McNamara	Sydney	1914	Place	82.0
B. Quinlan	Melbourne	1966	Drop	77.7
P. Vinar	Geelong	1965	Drop	76.8
R. Kercheaval	Chicago	1935	Punt	83.2
P. Vinar	Melbourne	1968	Punt	73.3

Table 1: Leading kicks made in kicking competitions, all using an Australian Rules football. From Atkinson[1]

The question remains as to how very long kicks, such as those shown in Table 1, are achieved. As well as the kicking–competition distances shown in this table, Atkinson [1] also includes kicks from practice and weekend matches, indicating that distances up to 98.5m have been achieved. Some of these are said to have included slight wind assistance. Measurements upon practice and match kicks are unlikely to be as reliable as official competition kicks, and therefore have not been included by us. An extraordinarily strong kick, or the perfect transmission of momentum to the ball, or an optimal orientation and spin of the ball in flight can all lead to a greater length of kick. These factors are difficult to quantify but conceptually a simple approach is to specify the initial angle and speed of projection, ignore rotation effects and treat drag as the main aerodynamic force. These are the assumptions that we will make in this paper, allowing us to treat the football as a projectile once it has left the foot of the kicker.

2. The Model

General aerodynamic principles Daish [5] indicate that for air flow past a football a reasonable first model for the drag force is to assume that it is proportional to the square of the speed of the ball. Then the governing equation for the trajectory of the football is

$$m\ddot{\mathbf{r}} = -mg\,\hat{\mathbf{j}} - {}^{1}/_{2}\,\rho A C_D\, v^2\,\hat{\mathbf{v}} \qquad (1)$$

with initial conditions $\mathbf{r} = \mathbf{0}$, $\dot{\mathbf{r}} = v_{ox}\hat{\mathbf{i}} + v_{oy}\hat{\mathbf{j}}$ at t=0. Here $\mathbf{r}$ denotes the position vector of the ball relative to its initial position at the time of being kicked, a dot denotes differentiation with respect to time, $\hat{\mathbf{i}}$ is a unit vector in the horizontal, $\hat{\mathbf{j}}$ is a unit vector vertically upwards, $\mathbf{v}$ is the velocity vector (where $\mathbf{v} = \dot{\mathbf{r}} = v\hat{\mathbf{v}}$), m denotes the mass of the football, ρ denotes the density of the air surrounding the football, A is a representative cross-sectional area of the ball, and C_D is the drag coefficient. The initial speed of propulsion v_o and the initial angle of propulsion to the horizontal α are related to the initial velocity components by $v_{ox} = v_o \cos\alpha$ and $v_{oy} = v_o \sin \alpha$. [See Fig. 1].

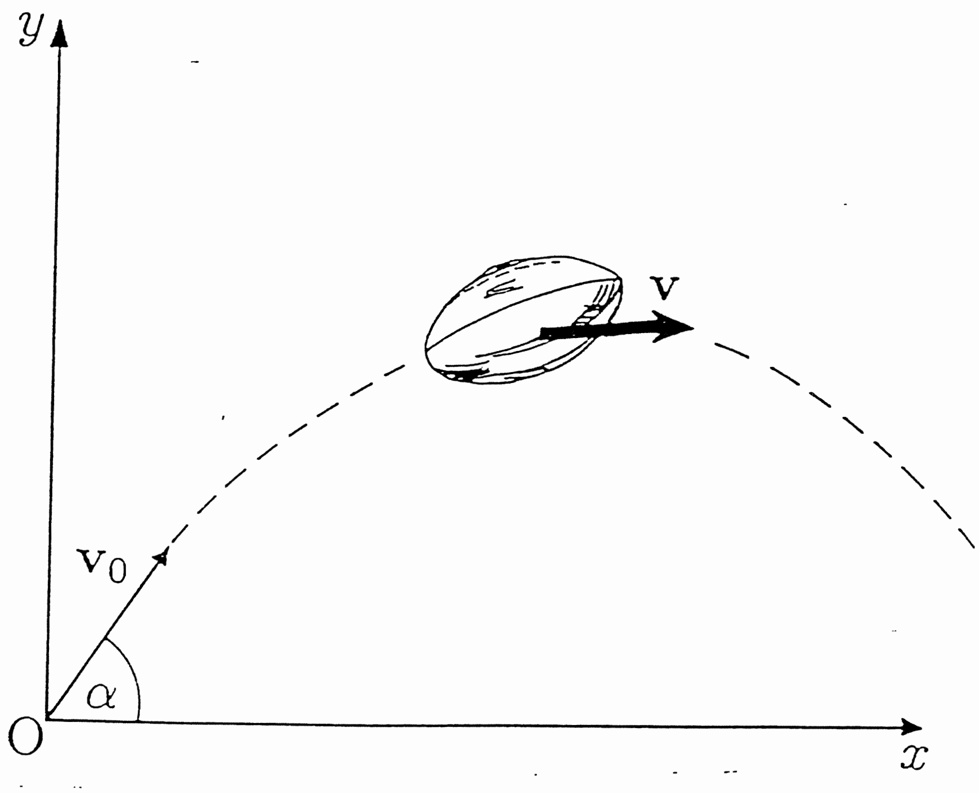

Figure 1. Co-ordinate system for the flight of a football

The modelling of a spherical soccer ball is simpler because the cross-sectional area is constant. The air flow past the ball produces in most cases a turbulent boundary layer with drag coefficient $C_D = 0.2$ as predicted by wind tunnel tests (see Daish [5]). Consequently the distance and time of flight to the first bounce could be calculated by numerical solution of the differential equations or semi-analytical techniques as proposed in de Mestre [6].

However, the ellipsoidal footballs used in other codes have different dimensions and shapes, and also may change orientation in flight. The only wind-tunnel information, that we are aware of, is due to Kerley [8] for an Australian Rules football which showed that the drag coefficient varies by a factor of four as the angle of attack changes. Kerley [8] also measured the lift force as a function of the angle of attack. The lift force in this case is due to an aerofoil effect rather than a rotational (Magnus) effect. He showed that the maximum magnitude of the lift force is approximately one–half of the magnitude of the drag force at angle of attack 45°.

To include the effect of lift, the equation of motion (1) must be appended by adding the lift force of magnitude $^1/_2\, \rho A C_L v^2$ in the direction perpendicular to the drag force. The lift coefficient C_L is an experimentally–determined quantity. In Cartesian coordinates, this results in the two differential equations:

$$m\ddot{x} = -\,^1/_2\, \rho A C_D \dot{x}\, (\dot{x}^2 + \dot{y}^2)^{1/2} - \,^1/_2\, \rho A C_L\, \dot{y}(\dot{x}^2 + \dot{y}^2)^{1/2} \quad , \tag{2}$$

$$m\ddot{y} = -\,mg - \,^1/_2\, \rho A C_D\, \dot{y}\, (\dot{x}^2 + \dot{y}^2)^{1/2} + \,^1/_2\, \rho A C_L\, \dot{x}(\dot{x}^2 + \dot{y}^2)^{1/2} \; . \tag{3}$$

3. Results

We were grateful when Dowell [7] presented us with the raw data upon which his 1976 paper with Cunningham was based. This consisted of initial velocity vector information and experimentally measured ranges for 31 kicks. The classifications used by Cunningham and Dowell [4] were as follows:

Type I kick, when the long axis of the football remains parallel to the path of the trajectory for most of the flight,

Type II kick, when the long axis of the football remains horizontal for the whole of the downward flight path,

Type III kick, when the long axis of the football remains at an angle to the horizontal so that the front tip of the ball remains ahead of and higher than the back tip for the whole of the downward flight path.

No comments are made for Types II and III about the orientation of the ball for the upward part of the flight path, but let us assume that their classifications hold for the whole of the flight path. For Type II the ball is then aligned with the tangent to the trajectory only near the top of the trajectory (the position of least speed for any trajectory), whereas for a Type III kick the ball is aligned with the tangent only near the beginning of the trajectory (the position of maximum speed). Therefore these classifications seem to correspond to minimum drag, maximum drag and intermediate drag respectively, which are different from the ideas put forward by Cunningham and Dowell [4] which were based upon an erroneous model. We will verify this assertion shortly.

Clearly other classifications are possible, such as the two proposed by Daish [5] consisting of the long axis closely aligned to the trajectory's tangent and the end–over–end tumbling mode. However it seems sensible to use Cunningham and Dowell's [4] classification, since we are using their data. Furthermore, Cunningham and Dowell [4] excluded tumbling or wobbling kicks, which would be even harder to model.

Our approach is to consider the first kick in each of these classifications and use a Runge – Kutta – Merson algorithm from the NAG library to numerically integrate the differential equation (1) to predict the combination of coefficients

$$k = \rho A C_D / (2m) \tag{4}$$

which exactly produced the observed range. The raw data (Dowell [7]) was provided in Imperial units but has been converted to metric for this paper. This modified data and the corresponding value of k for the three control kicks are shown in Table 2.

Type	$v_0(ms^{-1})$	α	Measured Range (m)	k
I	34.1	50°	57	0.0128
II	33.7	54°	47	0.0169
III	33.0	50°	49	0.0159

Table 2: Raw data yielding appropriate k values for kick Types I, II, III

In principle, one can calculate the drag coefficient C_D from the values for k given in Table 2 once the other parameters in equation (4) are known. However, in practice, the whole of k is the important parameter. Since for the three types of kicks ρ and m were fixed, and A was chosen as the minimum cross–sectional area of the ball, Table 2 supports our assertion that the drag for Type I is less than the drag for Type III which is less than the drag for Type II.

With the above values for k the equation (1) was again solved using the NAG library routine to predict the range and time of flight for the remaining 28 kicks. In Tables 3, 4 and 5 calculated times of flight are listed, and calculated and measured ranges are compared for Type I, Type II and Type III kicks respectively.

Kick Number	v_{ox} (ms^{-1})	v_{oy} (ms^{-1})	Measured Range (m)	Calculated Range (m)	Calculated Time of Flight (s)
2 (control)	21.90	26.09	56.69	56.64	4.24
5	24.05	23.63	55.78	58.16	3.89
6	20.67	24.63	53.95	53.17	4.08
9	20.37	27.03	55.78	54.43	4.38
15	24.12	25.87	54.86	60.40	4.17
17	21.70	24.96	57.60	55.34	4.10
21	22.52	25.01	48.46	56.86	4.09
24	20.27	27.90	52.12	54.80	4.49
27	19.98	27.50	56.69	53.97	4.45
29	20.85	25.75	62.18	54.42	4.22
32	23.49	27.02	54.86	60.24	4.33
39	21.89	24.31	53.95	55.11	4.01

Table 3: Predicted versus measured range for Type I kicks (k = 0.0128)

Kick Number	v_{ox} (ms^{-1})	v_{oy} (ms^{-1})	Measured Range (m)	Calculated Range (m)	Calculated Time of Flight (s)
4 (control)	19.80	27.25	46.63	46.64	4.21
7	22.94	24.60	42.97	50.10	3.85
10	22.80	26.70	52.12	51.15	4.09
11	22.11	26.35	50.29	49.91	4.07
13	21.64	25.79	52.12	48.88	4.01
16	19.99	24.68	54.86	45.64	3.91
19	19.66	26.09	42.06	45.87	4.08
28	19.42	24.86	43.89	44.82	3.95
30	17.92	22.13	43.89	40.66	3.64
33	23.66	25.37	54.86	51.64	3.92
36	19.54	24.12	52.12	44.58	3.85
40	20.33	24.23	53.03	45.91	3.85

Table 4: Predicted versus measured range for Type II kicks ($k = 0.0169$)

Kick Number	v_{ox} (ms^{-1})	v_{oy} (ms^{-1})	Measured Range (m)	Calculated Range (m)	Calculated Time of Flight (s)
3(control)	21.19	25.25	49.38	49.34	4.00
8	21.23	25.30	50.29	49.44	4.01
12	23.34	23.34	41.15	51.27	3.73
14	18.81	28.96	49.38	47.06	4.47
20	20.33	26.02	51.20	48.38	4.11
31	24.25	23.01	45.72	52.30	3.67
35	23.06	25.61	56.69	52.52	4.01

Table 5: Predicted versus measured range for Type III kicks ($k = 0.0159$)

Although 31 kicks were recorded they were numbered to 40, indicating that at least 9 kicks were discarded as raw data because of wobbling or tumbling during the experiments. Using the criterion that the model is suitable if the predicted range is within 10% of the measured range it is seen that 8 out of 11 Type I kicks, 7 out of 11 Type II kicks and 4 out of 6 Type III kicks are predicted satisfactorily, excluding the three control kicks in Table 2 (which are predicted exactly of course). Other control kicks could have been chosen, producing slightly different values for k and hopefully even better agreement within the above criterion. However the large number of successful predictions with the present arbitrary choice indicates that the model is reasonably satisfactory.

The distance obtained for each kick is about half of the distance predicted by the gravity-only formula $v_0^2 \sin 2\alpha/g$. This verifies that drag is quite important for a football kick, as shown in figure 2.

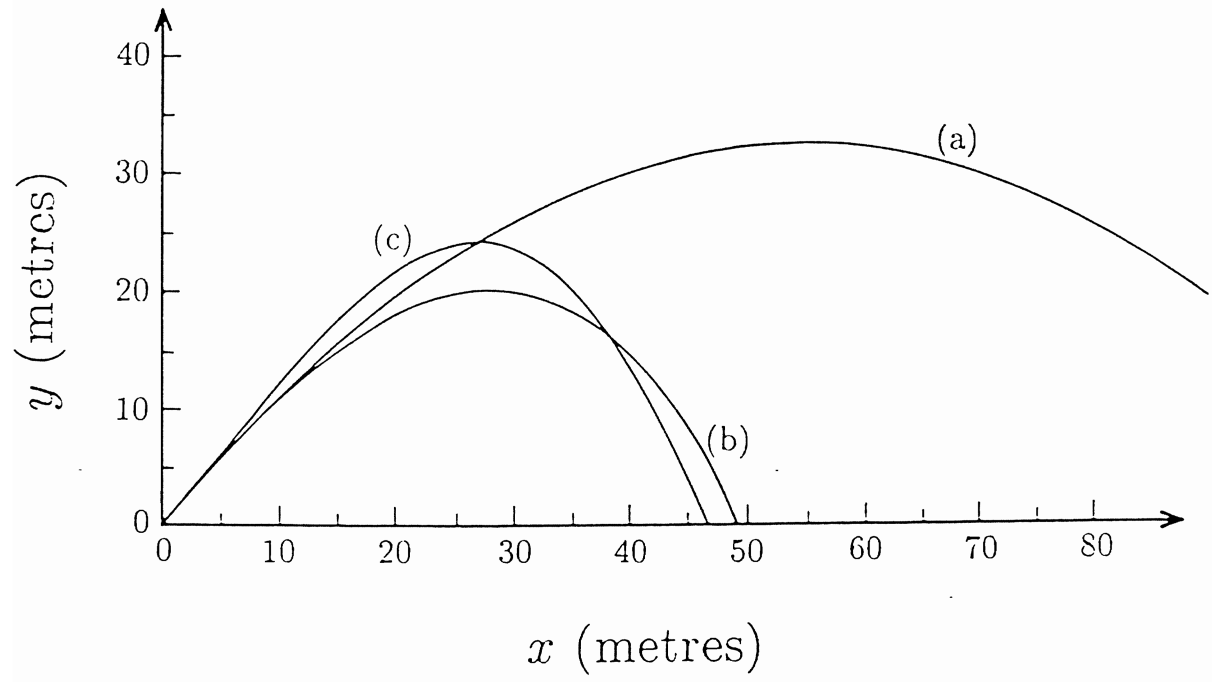

Figure 2. Calculated trajectories for the Type III control kick.

(a) Gravity only, no drag or lift.
(b) Gravity and drag, no lift.
(c) Gravity, drag and lift.

Further computations indicate that for v_0 and α typical of football kicks, the range is 2–3 times more sensitive to changes in k than is the time of flight. A 10% increase (decrease) in k produces a 5% decrease (increase) in range. The sensitivity was similar for all three types of kick. We believe that the sources of error in the experiments of Cunningham and Dowell [4] could easily add to an overall error of ± 10%. In the absence of a full error analysis by Cunningham and Dowell [4] our criterion for the validity of our model seems reasonable.

Another potentially important effect is lift, which can be due to either rotation (Magnus effect) or an "aerofoil" effect. For a spherical ball only the Magnus effect can contribute and depending upon the axis of rotation, can result in an upwards, downwards or sideways deflection (Daish [5]). An example of its application is a free kick in soccer which the talented kicker can bend around a wall of defenders and into the goal mouth.

For an ellipsoidal ball the Magnus effect results in no upwards or downwards force when the axis of rotation is in the plane of the trajectory of the ball. This is the case for all kicks of Types I, II and III. On the other hand, the "aerofoil" effect may generate vertical forces for these types of kicks.

The "aerofoil" effect occurs when the ellipsoidal ball is misaligned with the trajectory, and this occurs in kicks of Type II and III. This is the same mechanism which 'gives rise' to lift for an aeroplane wing. We tried to include this effect with equations (2) and (3) using $C_L = 0.5C_D$ as a representative upper bound, based upon the wind tunnel tests of Kerley [8]. The equations were solved numerically with the same NAG algorithm. For the control Type II kick we found that the range decreased by 10%, while the hang time increased by 14%. For the control Type III kick we found that the range decreased by only 5%, while the hang time increased by 16%. This gives us order of magnitude estimates of the effect of lift on the flight of a football, which seems to be quite important as far as the hang time is concerned.

4 . Further Investigations

There are many problems that still need to be addressed, some of them with the possible aid of this mathematical model.

(i) Wind effects could be calculated numerically using an analysis similar to de Mestre [6].

(ii) As mentioned in the introduction, Brancazio [2] was interested in obtaining a suitable balance between "hang time" in the air and distance kicked, so that followers could reach the receiver before he could run the ball back. A similar problem is encountered in both rugby codes. Brancazio [2] considered the gravity-only equation for which the maximum "hang time" is obtained at the minimum range (straight up and down) for a given kicker. When drag and lift are included it could be useful to consider the optimization problem of maximizing a quantity of the form

$$(\text{Range/Max. Range})^2 + (\text{Hang Time/Max. Hang Time})^2 \tag{5}$$

to identify suitable initial speeds and angles for such kicks.

(iii) It could also be useful to investigate the differences in the flight of a football set into motion by a place kick, a punt kick and a drop kick. For a soccer ball a punt kick or a drop kick seem to be the preferred methods of kicking to obtain maximum distance. From the data in Table 1 on official competition kicks of an Australian Rules ball it is unclear which method of kicking gains the maximum distance. One of us is still trying to achieve a "torpedo place kick".

(iv) Another problem that could be investigated is the idea that an around–the–corner run–up to a place kick is more accurate and stronger than a direct-attack run up. Is this to do with the biomechanics of the kicking action or is there some other reason?

(v) It is apparent that there is a need for more comprehensive wind tunnel studies on various types of footballs. This would permit a more thorough modelling of the flight of a football if drag and lift coefficients were determined. This needs to be done for a stationary football at various angles of attack and for a rotating football. Visualization of the flow around the football would also be interesting.

All these questions and the preceding analysis indicates that there are many interesting mathematical problems associated with the process of kicking a football. It remains to be seen whether or not the participants and coaches of the codes would like to know the theoretical answers, or whether they are satisfied with practical knowledge gained by trial and error and long-term experience.

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MATHEMATICS APPLIED TO SPORT

Neville de Mestre[1]

Abstract

Mathematics can be applied to sport at the level of high school mathematics or tertiary mathematics. Examples of the former are given in cricket, tennis, baseball and golf draws. Examples that need an understanding of university mathematics include the shielding of middle-distance runners, the design of a triathlon, the modelling of sporting projectiles, and an analysis of bat-ball collisions in cricket. Areas of sport that have problems which could benefit from a mathematically-based approach are pointed out.

1. Introduction

Mathematics can be used at various levels of sophistication in many sporting events. Most users are limited to the application of basic arithmetic in scoring, but a few are now applying higher mathematics to help them improve techniques, skills and (eventually) results.

The mechanics, or physics, of sports has received detailed attention in books by Daish [6] and Brancazio [1], which seem to be the best two books with this emphasis that I am aware of. Books with even more mathematical emphasis include Townend [11] and de Mestre [7].

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Two mathematically-related papers have recently appeared by Crothers [4], [5]. The first is a pot-pourri of sports that have some mathematical connection, and emphasizes the effect of spin. However only two problems are treated in any mathematical detail – the cueing of a snooker ball, and the re-emergence of a golf ball when it has rolled tangentially into the cup on a golf green. The second paper covers some optimization problems in high-jumping, rugby, squash and the triple jump. Some fluid flow problems are briefly mentioned, and a section is devoted to codes for passing hidden messages verbally to team-mates during play.

In this paper I intend to cover three areas relating to mathematics and sport. I will briefly mention some problems arising from sport that have a mathematical basis suitable for high school students. This is the next step up from the simple arithmetical scoring approach limited to the majority of sports participants. I will then concentrate on areas of mathematical research in sport that I have had experience with. Finally I will point out the areas of research that currently need some mathematical investigation.

2. High School Mathematics and Sport

Many of the problems, that mathematicians are consulted on, need only a basic understanding of high school mathematics. For this reason I thought that I would present a number of sporting problems that I include in my inservice course to high school teachers on "Mathematics and Sport".

Problem 2.1: Adams and Brown have each taken 28 wickets for 60 runs before the last match of the season is played. In the last match Adams takes 0 wickets for 24 runs, while Brown takes 5 wickets for 40 runs. Which bowler ends the season with the better average?

Answer: It comes as somewhat of a surprise that Adams has the better average (3 runs per wicket) compared with Brown who has a season average of 3.33 runs per wicket. The mathematics involved is elementary, but for general bowling figures for Adams and Brown the problem becomes an exercise in inequalities. This has all been investigated by Burns [3].

Problem 2.2: In a tennis match where the winner is decided from the best of three sets, what is the minimum proportion of points that a player can win and still win the match?

Answer: Suppose that the winner loses one set to love, and each game to love in this set. Then the winner has won no points and the loser has won 24 so far.

Now consider the other two sets all won by the winner eventually. Assume that each set is 7-6 with the tie-breaker being 7-5 in both cases. The losing 6 games in each set could score no points for the winner and 24 points for the loser. The winning 6 games in each set, other than the tie-breaker, could score 24 points for the winner and 12 points for the loser. Thus in each 7-6 set the winner will win 31 points and the loser will win 41 points. Therefore over the three sets, the winner wins 62 points and the loser wins 106 points giving a minimum proportion of 36.90%. Somewhat surprisingly the inclusion of games to deuce, or tie-breakers with larger scores, cannot give a proportion below this value.

Problem 2.3: In 1941 Joe DiMaggio set a most unusual record in baseball. He made at least one safe hit in each of 56 consecutive games. His batting average for the year was 0.357. If he batted 4 times per game for the whole 56 games, calculate the probability of this record being performed.

Answer: The probability of a safe hit each time at bat = 0.357

The probability of not getting a safe hit each time at bat = 0.643

The probability of not getting a safe hit in 4 times at bat $= (0.643)^4$

$= 0.171$

Therefore, the probability of at least one safe hit in 4 times at bat = 0.829

That is, the possibility of at least one safe hit in each game = 0.829

Hence, the probability of at least one safe hit in 56 consecutive games $= (0.829)^{56}$

$= 0.000027$

This makes it a pretty spectacular record! Mathematicians will notice the similarity of the solution to that for the "23-in-a-room" birthday problem.

Problem 2.4: Eight golfers are on a golfing safari for a week. Each day they wish to play foursomes golf, that is, in pairs against another pair. They want to arrange a draw so that they each partner everybody once, and play against everybody exactly twice.

Answer: The draw for playing with a different partner each day can be done by a cyclic permutation of 7 players keeping one person fixed. This can also be obtained geometrically as follows.

Let the players be numbered 1 to 8. Place No. 1 at the centre of a circle. Place the remaining seven players evenly around the circumference of the circle, that is, $51\frac{3}{7}$ degrees apart (Figure 1). Join 1 to 2, 8 to 3, 7 to 4, 6 to 5. This is the draw for round 1, if playing with a different partner each day was all that was needed. To get the draw for succeeding days, the straight line pattern would be rotated step-by-step around the circle. Thus the draw for round 2 would be 1 to 3, 2 to 4, 8 to 5 and 7 to 6. Thus the draw for round 2 would be 1 to 3, 2 to 4, 8 to 5 and 7 to 6. This solution is equivalent to the solution for the problem of making up a competition draw for 8 teams, so that each team plays each other once.

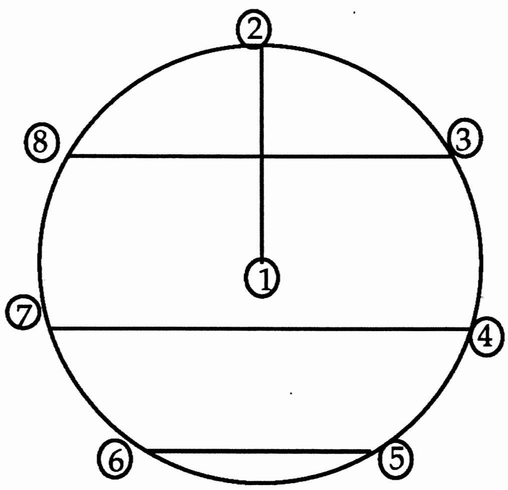

Figure 1. Draw pattern for 8 players playing with each other once.

The geometrical approach has advantages when it comes to solving the full golf foursomes problem which includes the constraint that each person must only oppose each of the other seven twice. The required pattern is shown in Figure 2.

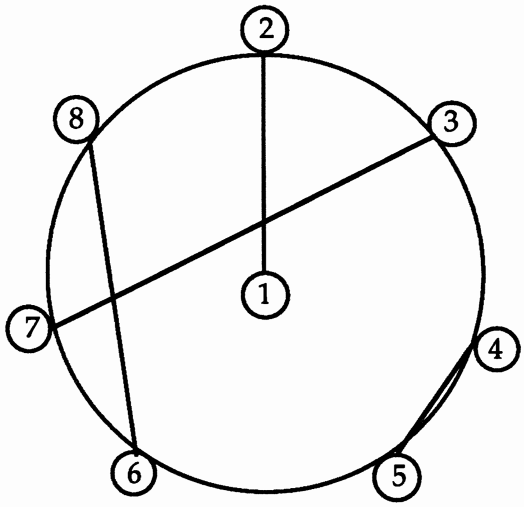

Figure 2. Draw pattern for 8 players playing with each other once, and against each other twice.

For the solution, it is important to introduce an asymmetrical pattern. Both constraints are then satisfied if the lengths of the chords are all different. The full draw is therefore

1-2	v	3-7	4-5	v	6-8
1-3	v	4-8	5-6	v	7-2
1-4	v	5-2	6-7	v	8-3
1-5	v	6-3	7-8	v	2-4
1-6	v	7-4	8-2	v	3-5
1-7	v	8-5	2-3	v	4-6
1-8	v	2-6	3-4	v	5-7

3. Past and Present Mathematical Research in Sport

Many of the current research projects involving mathematics and sport have been presented in other papers at this conference and elsewhere (see earlier references). I will therefore concentrate on a few sporting problems that I have attempted to solve using mathematics to indicate the variety available.

Problem 3.1: Determine the effect of being shielded by the front runner in a middle-distance foot race.

Solution: Experimental studies of human beings running on treadmills have shown that the power P_t developed by a runner is proportional to the speed V_t. Thus

$$P_t = kV_t$$

where k is a constant which varies from one runner to another, but is usually about 0.3 kilojoules per metre.

On the track the power P developed when running with speed V is given by

$$P = kV + \frac{1}{2}\rho AC_D V^3$$

where the extra term is the power used to overcome air resistance. The drag force $\frac{1}{2}\rho AC_D V^2$ due to air resistance contains the air density ρ, the runner's frontal area of cross-section A, and the drag coefficient C_D. For a typical middle-distance runner $\frac{1}{2}\rho AC_D \approx 0.216$, and since a typical speed is $6ms^{-1}$ the power lost to air resistance is approximately 47 joules sec^{-1}. The total power developed at this speed is therefore 1847 joules sec^{-1}, and so about 2.5% of this power is used to overcome air resistance.

Now when a runner chooses to be shielded by the front runner, several studies have shown that the aerodynamic drag force is reduced to between $\frac{1}{5}$ and $\frac{1}{2}$ of the drag force experienced by the front runner, depending on how close the trailing runner "can tuck in" behind the front runner. If we assume that the aerodynamic drag force for the shielded runner is $\frac{1}{4}$ of that for the front runner, then this makes the total power developed by the shielded runner in maintaining this speed only 1812 joules sec^{-1}. Thus he saves about 2% of his energy for the final sprint.

The effect (and subsequent savings) are even greater when there is a strong wind blowing parallel to the straight sections of the track.

Problem 3.2: Design a triathlon so that each of the three activities used (running, cycling, swimming) has an equal bearing on the outcome of the race.

Solution: A three-component triathlon is sought in which the proportions of running, cycling and swimming are such that the winner is the best triathlete. Certainly the event would be poorly-designed and the winner not the best triathlete if one activity was over an inordinately short time or length compared with the other two.

The most common distances for a triathlon seem to be those used in the last World Championship, namely 1.5km of swimming, 10 km of running and 40 km of cycling. To determine whether this proportion of distances S : R : C = 1:6.67:26.67 is the most suitable for predicting the best triathlete, various criteria on the mixing of the distances for each activity need to be looked at. Three criteria that could be used are:

(i) equal times for each activity

(ii) equal energy expended for each activity

(iii) equal spread of times over each activity for all competitors.

(i) The individual results for each section of the 1991 World Championship triathlon are used to prepare Table 1. Distance ratios are suggested on the basis of equal times for each activity.

	Swim Rate (ms^{-1})	Run rate (ms^{-1})	Cycle rate (ms^{-1})	S:R:C
Male winner	1.29	4.96	12.03	1:3.84:9.33
Fastest male	1.33	5.19	12.33	1:3.90:9.27
Female winner	1.14	4.39	10.72	1:3.85:9.40
Fastest female	1.16	4.55	10.80	1:3.92:9.31
Median male (258th)	1.04	4.27	10.42	1:4.11:10.02
Median female (145th)	0.85	3.67	9.74	1:4.32:11.45

Table 1: Swim, Run and Cycle rates for 1991 World Triathlon Championship

It is clear that on this basis the World Champions (male and female) should concentrate their training on running and cycling until the current imbalance is redressed.

(ii) If the criterion is that equal energy be expended on each section then a measure of these is needed during each activity. This is not easy to obtain directly, but certain measurements can be used which relate indirectly to the energy expended.

The main one is the amount of oxygen consumed, an indicator of aerobic-power output. Differences between individuals, because of weight and shape, are incorporated in the standard unit of energy expended when resting, defined as one metabolic unit (1 MET). The concept of METS does not compare one person with another, it compares the energy demands from one activity to another for the same person. The number of METS increases for an individual as the rate of any activity increases. From data in Fahey [10] it is seen that 18 METS are used for swimming at 1.04 ms^{-1}, running at 4.65ms^{-1} and cycling at 11.58ms^{-1}. These are rates near the median male's rates in the 1991 World Triathlon Championship, and therefore are a sound basis for calculation. They produce a proportion based on equal energy expended of S:R:C = 1:4.47:11.13.

(iii) Equal time spread of data from a triathlon race is based on the idea that a competitor who is strong in one section but only average in the other two should finish in the same place independently of which section he/she is strong in. That is, the spread of times over each activity should be the same. There are a number of ways to measure this spread but a very simple one is based on the difference in time between the fastest and the median competitor for each activity. For the 1991 World Championship this produced 324 seconds for the swim, 433 seconds for the run and 550 seconds for the cycle, and a suggested proportion of S:R:C = 1:4.99:15.71.

Whichever criteria is preferred, it is clear that the current triathlons are not designed properly to determine the best triathlete. It may be that the current world champion would still win even if the course was changed to 2 km swim, 10 km run, 30 km cycle agreeing closely with criterion (iii). Modifications to the distances based on criteria (i) or (ii) would be even more drastic.

Problem 3.3: Investigate the mathematical modelling of sporting projectiles.

Solution: Much of the research into this has been published or referenced in de Mestre [7]. Essentially, Newton's laws of motion are applied to produce a system of differential equations. The dependent variables are the position of the centre of mass of the projectile and the angles through which the projectile has been rotated during its flight; the independent variable is the time.

The simplest shape to consider for a projectile is a sphere, because of its symmetry, and because its drag in air is well documented. The human body as a projectile (long jumps, high jumps, diving and gymnastics) is a constantly-changing shape as far as air drag is concerned, and the drag coefficient is much more difficult to quantify.

When a symmetrical projectile is spinning or rotating a lot, the aerodynamic force of lift has to be included as well as the drag. Lift can act laterally (the hook or slice in golf) as well as vertically (the top-spin or bottom-spin in tennis). Lift is also important for a non-spinning asymmetrical projectile (Nordic ski jumper).

The main forces acting on a sporting projectile are gravity and the aerodynamic forces. The associated vector differential equation for the centre of the mass of the projectile is:

$$m \frac{d\underset{\sim}{v}}{dt} = m\underset{\sim}{g} - \frac{1}{2}\rho A C_D v^2 \hat{\underset{\sim}{\tau}} + \frac{1}{2}\rho A C_L v^2 \hat{\underset{\sim}{n}} + \frac{1}{2}\rho A C_s v^2 (\hat{\underset{\sim}{\tau}} \times \hat{\underset{\sim}{n}})$$

where m is the mass of the projectile, $\underset{\sim}{v}$ is its velocity vector, t is the time, $\underset{\sim}{g}$ is the gravity vector per unit mass, ρ is the density of air, A is the frontal area of the projectile, v is its speed, C_d, C_L and C_S are respective drag, lift and sideways coefficients, $\hat{\underset{\sim}{\tau}}$ is a unit vector in the tangential direction to the projectile's path, and $\hat{\underset{\sim}{n}}$ is a unit normal vector perpendicular to $\hat{\underset{\sim}{\tau}}$ and in the vertical plane. Usually C_D, C_L and C_S have to be determined from wind-tunnel tests, and they frequently vary with the speed. Even when they are averaged over a range of speeds, and modelled as being constant for that range, the exact solutions of the system of differential equations is not known analytically.

In some cases (the shot put and long jump) the aerodynamic forces are much smaller than the gravitational forces, and approximate solutions can be ascertained by a perturbation analysis. For most sporting projectiles however, the solution of the differential equations can only be obtained numerically, usually employing a Runge-Kutta package.

When rotation effects are included the extra differential equations are incorporated in

$$\frac{d\underset{\sim}{H}}{dt} = \underset{\sim}{L}$$

where $\underset{\sim}{H}$ is the moment of momentum about the centre of gravity and $\underset{\sim}{L}$ is the moment of the aerodynamic forces about the centre of gravity.

Thus when the aerodynamic forces act through the centre of gravity and the initial spin velocity is zero, the projectile doesn't spin at all (the shot put), but if asymmetry of the projectile results in aerodynamic forces not acting through the centre of mass of the projectile then the projectile will continually change its orientation along the flight path (rugby or Aussie Rules footballs). Calculations for the latter situation can be extremely complicated, and can only be obtained by numerical methods.

A further complication to projectile trajectories is the presence of wind. This dramatically affects the aerodynamic forces, which depend on the speed of the projectile relative to the moving air mass. This again adds complexities to the numerical calculations. One successful inclusion of wind effects has recently been accomplished by de Mestre [8] for the case of a long-jumper.

Problem 3.4: Analyse bat-ball collisions in cricket.

Solution: A special frame was constructed to allow a cricket bat to rotate about a horizontal axis through the handle. A cricket ball was also suspended within the frame, which allowed the bat-ball impact to take place at various controlled positions along the face of the bat.

The position of the centre of percussion (sweet spot) and the point of impact for maximum speed of the ball off the bat were determined both theoretically and experimentally. For details see Brearley, Burns and de Mestre [2]. The centre of percussion predictions were confirmed, but the impact for maximum speed was not. It appears that further theoretical investigations may need to take account of the elasticity of the bat, and not treat it as a rigid body.

4. Future Mathematical Research in Sport

There are a number of projects that currently need to be analysed using mathematics or mathematical computing. One of the most useful would be to develop a computer graphics package of the flight of a spinning spherical projectile, so that a real-time image of an approaching or receding ball as seen by the batter (or golfer) can be depicted on the screen. This would have an immediate application in cricket as it could be adapted to help with an l.b.w. appeal. If the batsman was removed and the characteristics of the ball's flight were known, the package could be used to see if the ball would have actually struck the wicket in the absence of the batsman.

There is also a need to construct a low speed wind tunnel with a high working section. Then the aerodynamic coefficients C_D, C_L and C_S could be measured for various athletes and balls in different orientations.

The National Sports Research Centre of the Australian Sports Commission publishes a booklet (Draper [9]) each year on the research needs identified by National sporting bodies in Australia. The current list includes the following problems which could be amenable to a mathematical or computational investigation.

(i) A comparison of the recurve, compound and cam bow from a biomechanical aspect.

(ii) An analysis of flat bottom or displacement malibu paddle boards to determine which shape is better suited in flat surf, choppy surf or big surf.

(iii) An investigation of training methods in the five pentathlon technical skills to determine the best preparation for optimum competition performance.

(iv) A determination of the effect of water temperature and wind speed on rowing performance.

(v) An investigation of the location of individuals and the type of rigging for multi-person rowing shells.

(vi) An analysis of the efficiency and effectiveness of round-the-corner place kicking and drop kicking in rugby.

This list is not exhaustive, as the workshop sessions at this conference verify. It merely emphasises that there is still plenty of scope for the application of mathematics to sport.

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